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ELECTRO-SPARK EXTRUDING

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REPUBLIC AVIATION CORPORATION
MANUFACTURING RESEARCH

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The propagation of pressure pulse phenomena and their ensuing effects are applied to the extrusion billet-die interface and pressure vessel container design by the formulation of a general solution derived from unsteady flow considerations. Specific application is pointed up in pressure vessel design by the assumption of reasonable postulates and application of them to solution of the problem.

BASIC INDUSTRY BRANCH
MANUFACTURING TECHNOLOGY LABORATORY

AERONAUTICAL SYSTEMS DIVISION
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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ABSTRACT-SUMMARY
Interim Technical
Progress Report No. 2

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FOREWORD

This Interim Technical Progress Report covers the work performed under Contract AF33(657)-11265 from October 1, 1963 to January 1, 1964. It is published for technical information only and does not necessarily represent the recommendations, conclusions or approval of the Air Force.

This contract with Republic Aviation Corporation of Farmingdale, New York, was initiated under ASD Manufacturing Technology Laboratory Project 8-111, "Electro Spark Extruding." It is administered under the direction of Mr. T.S. Felker of the Basic Industry Branch MATB, Manufacturing Technology Laboratory, Aeronautical System Division Wright-Patterson Air Force Base, Ohio.

Mr. J.H. Wagner of the Manufacturing Research Department, Republic Aviation Corporation is the engineer in charge of the project. Mr. Gunther Pfanner is cooperating in the research.

The work presented in this report was done by B. P. Leftheris of Re-entry Simulation Laboratory, Research Division.

The primary objective of the Air Force Manufacturing Methods Program is to increase producibility, and to improve the quality and efficiency of fabrication of aircraft, missiles and components thereof. This report is being disseminated in order that methods and/or equipment developed may be used throughout industry, thereby reducing costs and giving "MORE AIR FORCE PER DOLLAR."

Your comments are solicited on the potential utilization of the information contained herein as applied to your present or future production programs. Suggestions concerning additional manufacturing methods development required on this or other subjects will be appreciated.

PUBLICATION REVIEW

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CONTENTS

Section		Page
INTRODUCTION		1
NOMENCLATURE		4
PART 1 - PULSE TRANSMISSION AND BILLET EXTRUSION		7
I	WAVE INTERACTION AT INTERFACE	8
II	EXTRUSION	10
III	EXAMPLE OF PULSE EXTRUSION	17
IV	DISCUSSION	19
PART 2 - HIGH PRESSURE TRANSIENT LOADS IN HOLLOW CYLINDERS		21
I	PROBLEM DESCRIPTION	22
II	PROBLEM FORMULATION	24
III	BOUNDARY CONDITION	32
IV	ENERGY CONSIDERATION	34
V	SOLUTION OF EQUATION 21	35
VI	EXAMPLE	45
VII	DISCUSSION	60
VIII	CONCLUSIONS	62
IX	FUTURE WORK	64
REFERENCES		65

INTRODUCTION

Capacitor discharge extrusion is a metal fabrication process involving application of the effects resulting from a rapid discharge of capacitor stored electrical energy to a billet in order to produce work in the form of an extrusion. The electro-hydraulic reduction of an extrudable billet material may be accomplished in a number of ways. One method is to apply the shock-pressure energy obtained from the capacitor discharge event directly to a billet in an ultra-high pressure fluid environment. Rapid gap discharge of electrical energy in an appropriate chamber causes a shock pressure wave to strike the billet face and, upon reflection, to create a dynamic force when added to the latent hydro-static force is sufficient to effect movement of the billet. The capacitor discharges are intended to produce a series of extremely short duration intense pressures considerably in excess of any pressures obtained hydrostatically in mechanically available in tooling of comparable size. It is believed that the short duration of the pressure pulses will allow a superposition of pressure without the corresponding increase in the design strength of the container by conventional hydrostatic criteria.

With the billet in the high hydrostatic pressure medium, an additional phenomenon of the capacitor discharge event is available for extrusion work; namely, the electromagnetic effects derived from the rapid discharge of electrical energy through an appropriately constructed conductive coil. These coil discharge phenomena may be applied by induction to the face of a piston-disc interposed between the coil and the billet to produce a shock wave in the high hydrostatic pressure environment from the rapid acceleration and sudden stop of the disc when the current ceases to flow. In addition, the electromagnetic repulsion may be applied to the billet face directly in order to realize the acceleration forces required to extrude the billet.

A further method of capacitor stored energy utilization is to

recover the energy created by the electro hydraulic discharge and to transfer it by mechanical means to the billet face. Displacement of the billet is accomplished mechanically by the use of a piston-ram on the billet material in a suitable container with the pressure pulse energy created by a series of discharges impinging on the face of the piston causing it to move forward to extrude the billet.

The advantages offered by the foregoing approach to electro-hydraulic extrusion are; (1) elimination of billet container wall friction, (2) reduction of billet die surface friction, (3) creation of pressures considerably greater than hydrostatically obtainable allowing extrusion ratios beyond those obtained hydrostatically, (4) reduced container size and cost.

Recognizing the attractive advantages of the application of this technology, the Aeronautical Systems Division of Wright Patterson Air Force Base has awarded Contract No. AF33(657)-11265 to Republic Aviation Corporation to determine the production potential for extruding steel alloys by capacitor discharge energy. This will be accomplished in the course of the program by conducting experiments with capacitor discharge equipment to develop the most suitable techniques for extruding steel sections. The program consists of two phases as follows:

Phase I - Design and Extrusion of the Equipment
Phase II - Development of the Extruding Process

The methods heretofore described will be investigated for suitability to realize the objectives of the program of extruding a 2 inch diameter billet to a 1/4 inch round.

This second quarterly report is concerned with the consequences of the explosive effects both in a hydrostatic medium and also in pressure vessel walls of a rapid discharge of electrical energy across a gap. Analyses have been prepared and original equations of state derived to consider the effects of shock pressure wave phenomena as

they will be applied in this program.

As the initial experiments place the billet in an ultra-high pressure fluid environment in order to realize the advantages derived from the hydrostatic extrusion process, the capacitor stored electrical discharges will be used to furnish the additional energy required to approach the target extrusion ratios of the program. For this reason examination of the new parameters extant becomes vital in order to understand the extrusion process at the billet die interface to predict ideally to what degree extrusion may occur; and, to what extent the extrusion process can logically be optimized from the variables involved in the electrohydraulic approach. This work is presented in the first section of the report.

Although pressure vessels are being safely used at hydrostatic pressures to 300,000 psi using well known standard static design criteria, the solution of the problem of containing similar pressures in a vessel with a superposed shock-pressure pulse loading was nowhere readily available for application to the equipment required to attain the objectives of this program. For this reason, an original analytical approach was pursued that has resulted in elastic-plastic equations of state that take into consideration the effects of shock loading in ultra-high hydrostatic pressure equipment. These derived criteria have been applied to the boundary conditions imposed on the container design by the electrohydraulic extrusion experiments with the conclusion that a container design that will accomplish the objectives of the program is feasible. This work is presented in the second section of this report.

NOMENCLATURE

Part 1:

ρ = density

C = acoustic speed

P = pressure

u = particle velocity

r = radius

x = distance

t = time

θ = angle

F = frictional force

N = normal force

μ = coefficient of friction

m = mass

ϵ_1 = axial strain

r_o = initial radius

x_o = initial length

v = Poisson's ratio

f_y = yield stress

NOMENCLATURE (cont'd.)

Part 2:

P	=	internal pressure
σ_1	=	radial stress
t	=	time
x	=	distance
r	=	radius during the wave action
u	=	particle velocity
σ_2	=	hoop stress
ρ	=	density during the wave action
ρ_o	=	density before the arrival of the wave
r_o	=	radius before the arrival of the wave
ϵ_1	=	radial strain
ϵ_2	=	hoop strain
E	=	$\frac{d\sigma}{d\epsilon}$
μ	=	Poisson's ratio
σ_y	=	yield stress from uniaxial tests
E_o	=	Young's modulus of elasticity
E_p	=	plastic modulus in the bilinear model
Φ	=	shift rate (propagation of the wave front)
C	=	acoustic speed
C_o	=	acoustic speed in the undisturbed state
V_o	=	propagation velocity of the wave front at $t = t_o$
w_1	=	propagation velocity of the wave at $r_o = a$

ζ	=	$\frac{\rho}{\rho_o}$
$\bar{t}$	=	$\frac{t \cdot C_o}{r_o}$
$\bar{u}$	=	$\frac{u}{C_o}$
$\bar{C}$	=	$\frac{C}{C_o}$
$\bar{\sigma}_1$	=	$\frac{\sigma_1}{E_o}$
$\bar{E}$	=	$\frac{E}{E_o}$
$\bar{\sigma}_2$	=	$\frac{\sigma_2}{E_o}$
$\bar{\sigma}_y$	=	$\frac{\sigma_y}{E_o}$
ω	=	$\sqrt{\bar{\sigma}_y \cdot \zeta}$

GENERAL DISCUSSION

From previous considerations of pulse extrusion, it was found that a pulse, generated by the discharge of a capacitor bank at one end of a liquid (preferably water) filled cylinder, a pulse of 215, 000 psi is necessary to achieve efficient extrusion. It was not, however, proven theoretically that such extrusion is possible, nor that a cylindrical container can be built to withstand the proposed high pressures.

The work in this report, therefore, is concerned with these two problems exclusively. The propagation of a pressure pulse and its ensuing effects require a different theoretical treatment from the existing treatments of flow (or extrusion), mainly because of their short duration: in gas dynamics, pressure waves represent unsteady flow as compared with the steady flow of fluids in pipes under a steady pressure from a pump.

The formulation of unsteady flow dynamics has been developed extensively[1, 2]. It is not, however, a simple matter to apply it because, for each case (with a given geometry and a given medium), the problem must be formulated anew.

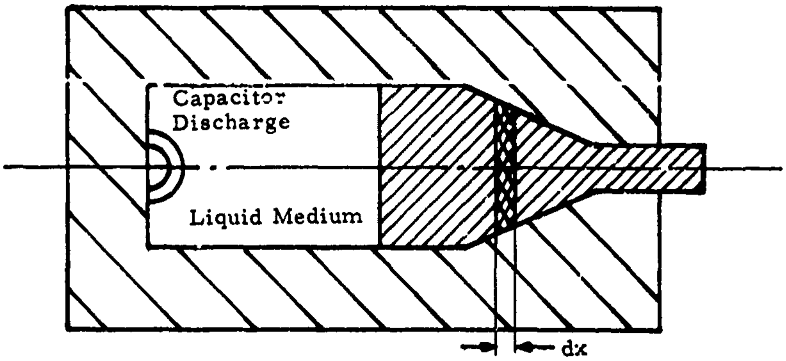
This work is divided into the following two parts:

Part 1 Pulse Transmission and Billet Extrusion
Part 2 High Pressure Transient Loads in a Hollow Cylinder

Both parts are generally formulated and general solutions are given and discussed. In each part, however, particular solutions are given with assumptions that this contractor considered to be reasonable. The results are sensible and, from the assumptions made, conservative. Nevertheless, it is hoped that the reader will form his own opinions about the solutions until further experimental proof is provided.

PART I - PULSE TRANSMISSION AND BILLET EXTRUSION

The general case for pulse extrusion is as follows: the front part of the cylindrical billet to be extruded fits into the tapered section as shown in the sketch: static pressure is then axially applied at the larger section through the liquid medium.

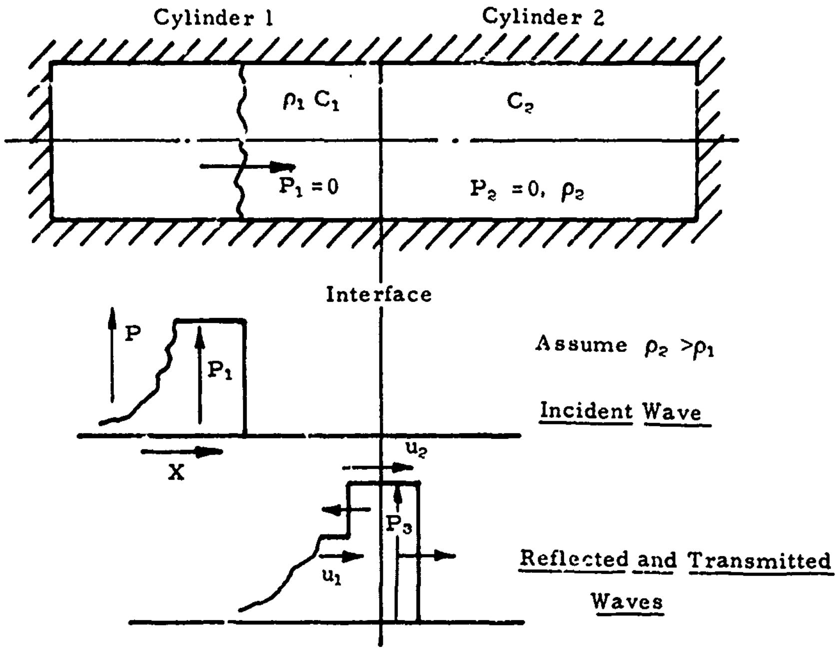


The capacitor discharge takes place on the left end of the cylinder and propagates to the right. Upon its arrival on the liquid-metal interface, it is partially transmitted and partially reflected.

In Part I, the wave interaction at the interface and the extrusion through the tapered section are theoretically analyzed.

SECTION I - WAVE INTERACTION AT INTERFACE

Consider two cylindrical bodies of the same diameter which are axially in firm contact with one another. Consider also a stress wave travelling along the axis towards the interface. It is required to find the strength of the reflected and transmitted waves.



Two conditions are necessary during reflection:

(1) The velocities of the wave on either side of the interface must be equal

(2) The pressures on either side of the interface must be equal

From momentum considerations immediately after reflection, we

obtain

$$\rho_1 C_1 (u_1 - u_2) = P_3 - P_1 = \rho_2 C_2 u_2 - \rho_1 C_1 u_1 \tag{1}$$

Hence

$$P_3 = \rho_2 C_2 u_2 = \frac{2 \rho_1 C_1 u_1}{1 + \frac{\rho_1 C_1}{\rho_2 C_2}} \tag{2}$$

$$\text{or} \quad P_3 = \left(\frac{2}{1 + \frac{\rho_1 C_1}{\rho_2 C_2}} \right) P_1 \tag{3}$$

It is obvious that when $\rho_1 . C_1 = \rho_2 . C_2$, $P_3 = P_1$ (i.e., there is no reflection). If, on the other hand, $\rho_1 . C_1 << \rho_2 . C_2$, then $P_3 = 2P_1$ (close end reflection).

The reflected wave is given by $(P_3 - P_1)$. Hence,

$$(P_3 - P_1) = \frac{\rho_2 C_2 - \rho_1 C_1}{\rho_2 C_2 + \rho_1 C_1} \tag{4}$$

The transmitted wave on the other hand is given by $P_3 = \rho_2 u_2 C_2$. Substituting from eq. (3) we have

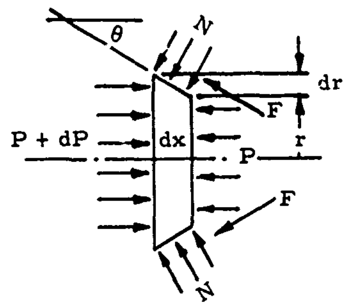
$$u_2 = \frac{2\rho_1 C_1 u_1}{\rho_2 C_2 + \rho_1 C_1} = \frac{2u_1}{1 + \frac{\rho_2 C_2}{\rho_1 C_1}} \tag{5}$$

Thus, if the amplitude and particle velocity of the incident wave are known, the amplitudes and particle velocities of the reflected and transmitted waves can be calculated.

SECTION II - EXTRUSION

The transmitted wave at the interface propagates through the billet towards the tapered section. It is now assumed that the die is made of the same material as the billet. There is, therefore, no reflection as the wave passes through the tapered section.

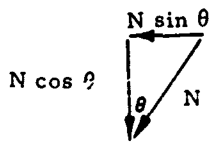
Consider an element of thickness dx which moves under the influence of the passing wave.

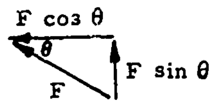


$$\frac{dr}{dx} = \tan \theta$$

From the momentum equation, we have $\Sigma F = m \frac{du}{dt}$ (6)

where, by definition, $u = \frac{\partial x}{\partial t}$





Also $F = N\mu$ (7)

$$\Sigma F \overset{+}{\rightarrow} (P + dP)(r + dr)^2 \pi - Pr^2 \pi - 2\left(r + \frac{dr}{2}\right) \pi \, dr \, (N\mu \cot \theta + N \sin \theta)$$

where $F = \mu \frac{N \, dr}{\sin \theta}$

The mass of the element is given by

$$dm = \pi \left(r + \frac{dr}{2}\right)^2 dx \, \rho$$

$$= \pi \left(r + \frac{dr}{2}\right)^2 \frac{dr}{\tan\theta} \rho$$

Thus the equation of momentum can now be written

$$2\pi \left(r + \frac{dr}{2}\right) \frac{dr}{\tan\theta} \rho \frac{du}{dt} = (P + dP)(r + dr)^2\pi - Pr^2\pi - 2\left(r + \frac{dr}{2}\right) \pi\, dr$$

$$(N\mu \cot\theta + N\sin\theta)$$

Transposing and neglecting second order terms, we have:

$$\frac{du}{dt} = 2 \frac{P}{\rho r} + \frac{1}{\rho} \frac{dP}{dr} - 2 \frac{N}{\rho r}(\mu \cot\theta + \sin\theta) \qquad (8)$$

We now consider mass continuity in the following manner. Consider an element initially of radius r_0 at position x_0, of thickness dx_0 and density ρ_0. Under the influence of the wave, the element moves to r, at position x and its thickness changes to dx; its density also changes to ρ.

Thus mass continuity is expressed as follows:

$$\pi r^2_0\, dx_0\, \rho_0 = \pi r^2\, dx\, \rho \quad \text{or} \quad r^2_0\, dr_0\, \rho_0 = r^2\, dr\, \rho$$

$$\frac{dx}{dx_0} = \frac{dr}{dr_0} = \frac{r^2_0\, \rho_0}{r^2\, \rho} \qquad (9)$$

We can now write $\frac{dP}{dr} = \frac{\partial P}{\partial r}$ since we considered only one instant of time, and $\frac{du}{dt} = \frac{\partial u}{\partial t}$ since we considered only one position. Similarly, $\frac{dr}{dr_0} = \frac{\partial r}{\partial r_0}$ for different times. Hence,

$$\frac{\partial u}{\partial t} = 2 \frac{P}{\rho r} + \frac{1}{\rho} \frac{\partial P}{\partial r} - \frac{N}{\rho r} B \qquad (10)$$

where $B = \mu \cot\theta + \sin\theta$

and $$\frac{\partial x}{\partial x_0} = \frac{\partial r}{\partial r_0} = \frac{r^2_0}{r^2} \frac{\rho_0}{\rho} \qquad (11)$$

Differentiating (6) with respect to t, we obtain

$$\frac{\partial u}{\partial r_0} = -(r_0^2 \rho_0) \frac{1}{(r^2\rho)^2} \frac{\partial(r^2\rho)}{\partial t}$$

The linear strain in the x direction can be written as follows:

$$\epsilon_1 = \frac{\frac{\partial x}{\partial x_0} dx_0 - dx_0}{dx_0} = \frac{\partial x}{\partial x_0} - 1$$

Hence,

$$\frac{\partial x_0}{\partial x} = \epsilon_1 + 1 \tag{12}$$

Combining (6) with (8), we obtain the following:

$$\frac{\partial x}{\partial x_0} = \frac{\partial r}{\partial r_0} = (\epsilon_1 + 1) = \frac{r_0^2}{r^2} \frac{\rho_0}{\rho} \tag{13}$$

Hence,

$$\frac{\partial \epsilon_1}{\partial t} = - \frac{(r_0^2 \rho_0)}{(r^2 \rho)^2} \frac{\partial (r^2 \rho)}{\partial t} \tag{14}$$

Thus

$$\frac{\partial u}{\partial r_0} - \frac{\partial \epsilon_1}{\partial t} = 0 \tag{15}$$

From (9) we can write

$$\frac{1}{\partial r} = \frac{r^2 \rho}{r_0^2 \rho_0} \frac{1}{\partial r_0}$$

Thus (5) can now be written as follows:

$$\frac{\partial u}{\partial t} - \frac{r^2}{r_0^2 \rho_0} \frac{\partial P}{\partial r_0} - \frac{2}{\rho r} \left\{ P - B N \right\} = 0 \tag{16}$$

If the strain in the two directions is restrained, the strain in the axial direction is given by:

$$\epsilon_1 = \frac{P}{E} - 2v \frac{P}{E}$$

Thus

$$\frac{\partial \epsilon_1}{\partial r_0} = \frac{(1-2v)}{E} \frac{\partial \rho}{\partial r_0}$$

Substituting in eq. (16), we obtain

$$\frac{\partial u}{\partial t} - \frac{r^2}{r_o^2 \rho_o} - \frac{E}{(1-2\nu)} - \frac{\partial \epsilon_1}{\partial r_o} - \frac{2}{\rho}\frac{f_y}{r} \quad (2 - B) = 0 \qquad (17)$$

Where $P = 2fy$ and $N = fy$ substitutions were made.

> NOTE: The billet is compressed continually under static pressure so that upon the arrival of the pulse, the material begins to flow plastically. The total available energy, therefore, is given by: a) the static fluid pressure which, in the case of a plasticized solid can be equal to twice the yield stress and b) the kinetic energy which can be found from the momentum equation as follows:
>
> From the momentum equation $(\Delta P) = \rho_o u w$
>
> where
>
> $$w = \frac{dr}{dt} = \frac{\partial r}{\partial t} + \left(\frac{\partial r}{\partial r_o}\right) \frac{dr_o}{dt}$$
>
> if, however, $\frac{dr_o}{dt} = o$, then $w = \frac{dr}{dt} = u$
>
> Hence $(\Delta P) = u^2 \rho_o$
>
> Thus, at any time, $\frac{(\Delta x) A}{2} . \Delta P = \frac{(\Delta x) A}{2} . \rho_o u^2$
>
> where A = Cross-sectional area
>
> Δx = An element of length
>
> Finally, $\frac{\Delta P}{\rho_o} = u^2$

Equations (15 and (16) form a system of partial differential equations. In order to solve them simultaneously, the method of characteristics will be used. The two general equations of characteristics of two dependent and two independent variables are as follows:

$$A_1\left(\frac{\partial U}{\partial X}\right)_Y + B_1\left(\frac{\partial U}{\partial Y}\right)_X + C_1\left(\frac{\partial V}{\partial X}\right)_Y + D_1\left(\frac{\partial V}{\partial Y}\right)_Y + E_1 = 0$$

$$A_2\left(\frac{\partial U}{\partial X}\right)_Y + B_2\left(\frac{\partial U}{\partial Y}\right)_X + C_2\left(\frac{\partial V}{\partial X}\right)_Y + D_2\left(\frac{\partial V}{\partial Y}\right)_X + E_2 = 0$$

where A_1, A_2, B_1, B_2, C_1, C_2, D_1, D_2, and F_1, F_2 are functions of U, V, X and Y.

Let
$$U = u$$
$$V = \epsilon_1$$
$$X = r_o$$
$$Y = t$$

Now rewriting (15) and (17) in characteristic form, we obtain

$$\left(\frac{\partial u}{\partial r_o}\right) + 0\left(\frac{\partial u}{\partial t}\right) + 0\left(\frac{\partial \epsilon_1}{\partial t}\right) - \left(\frac{\partial \epsilon_1}{\partial t}\right) = 0$$

$$0\left(\frac{\partial u}{\partial r_o}\right) + \left(\frac{\partial u}{\partial t}\right) - \frac{r^2}{r_o{}^2 \rho_o}\ \frac{E}{(1-2v)}\left(\frac{\partial \epsilon_1}{\partial r_o}\right) + 0\left(\frac{\partial \epsilon_1}{\partial t}\right) - \frac{2 f_y}{\rho_o r}\ (2\text{-}B) = 0$$

where

$$A_1 = 1 \qquad A_2 = 0$$
$$B_1 = 0 \qquad B_2 = 1$$
$$C_1 = 0 \qquad C_2 = -\frac{r^2}{r_o{}^2\ \rho_o}\ \frac{E}{(1-2v)}$$
$$D_1 = -1 \qquad D_2 = 0$$
$$E_1 = 0 \qquad E_2 = -\frac{2 f_y}{\rho_o r}\ (2 - B)$$

The solution of the system is given as follows:

$$\Phi = \frac{dt}{dr} = \mp\sqrt{\frac{r^2}{r_o{}^2 \rho_o}\ \frac{E}{(1-2v)}} \quad - \frac{r^2}{r_o{}^2 \rho_o}\ \frac{E}{(1-2v)}$$

$$= \mp\left(\frac{r}{r_o}\right)\sqrt{\frac{E}{\rho_o}\ \frac{1}{(1-2v)}} \quad - \frac{r^2}{r_o{}^2 \rho_o}\ \frac{E}{(1-2v)} \tag{18}$$

$$\therefore du \mp \left(\frac{r}{r_o}\right)\sqrt{\frac{E}{\rho_o}\ \frac{1}{(1-2v)}}\ d\epsilon_1 - \frac{2}{\rho r}\ (2\text{-}B)\ f_y\ dt = 0$$

and
$$\frac{du}{dt} \mp \left(\frac{r}{r_o}\right)\sqrt{\frac{E}{\rho_o}\ \frac{1}{1-2v}}\ \frac{d\epsilon_1}{dt} - \frac{2 f_y}{\rho r}\ (2\text{-}B) = 0 \tag{19}$$

The partial differential equations are replaced, therefore, by two total differential equations.

Now let $\rho = \rho_o$ (constant during plastic extrusion).

Hence $$\epsilon_1 + 1 = \left(\frac{r_o}{r}\right)^2 \tag{20}$$

We shall, furthermore, assume that extrusion takes place in the plastic region with $E = 0$

Equation (15) can therefore be written

$$\frac{du}{dt} - \frac{2f_y}{\rho_o r} (1-B) = 0 \tag{21}$$

but $r = r(r_o, t)$

Hence

$$\frac{dr}{dt} = \left(\frac{\partial r}{\partial t}\right) + \left(\frac{\partial r}{\partial r_o}\right) \frac{dr_o}{dt}$$

$$= u \mp \frac{r^2_o}{r^2} \cdot \frac{r}{r_o} \sqrt{\frac{1}{\rho_o} \frac{E}{1-2v}}$$

$$= u \mp \frac{r_o}{r} \sqrt{\frac{1}{\rho_o} \cdot \frac{E}{(1-2v)}}$$

If $E = 0$, $$\frac{dr}{dt} = \frac{\partial r}{\partial t} = u \tag{22}$$

Thus equation (16) becomes

$$\frac{d^2 r}{dt^2} = \frac{2f_y (2-B)}{\rho_o} \quad \frac{1}{r} \tag{23}$$

A familiar and exact equation can be derived by rewriting eq. (23) as follows.

$$\frac{dr}{dt} \, d\left(\frac{dr}{dt}\right) = \frac{2(2-B) f_y}{\rho_o} \frac{dr}{r}$$

or

$$\int_o^{u_o} du^2 = \frac{2(2-B)f_y}{\rho_o} \int_r^{r_o} \frac{dr}{r}$$

Hence

$$u_o^2 = \frac{4(2-B)f_y}{\rho_o} \ln \frac{r_o}{r} \tag{24}$$

Equation (24) is similar to the steady-state extrusion. In fact, it is identical to the potential energy replaced by the kinetic energy.*

Since u_0^2 is a positive quantity, we deduce that $0<B<1$ If we choose $B = 0$ (for $\mu = .5$, $\theta = 39^o$), then we have

$$u_0^2 = \frac{8f_y}{\rho_0} \ln \frac{r_0}{r} \qquad (25)$$

* The mechanism of the propagation of the wave is as follows: a) the wave is propagated partly with an elastic steep front behind which the velocity of the particles is u_0, and b) immediately afterwards, the decelerating process takes place: the propagation velocity of the deceleration may be zero if $E = 0$ (i.e., the flow is plastic).

SECTION II - EXAMPLE OF PULSE EXTRUSION

Let $P_1 = 215,000$ and $f_y = 90,000$ psi (f_y = the yield strength of annealed 4340 steel).

Hence, from eq. (5)

$$u_2 = \frac{2P_1}{\rho_1 C_1 + \rho_2 C_2}$$

where conditions 1 (subscript 1) refer to water and conditions 2 (subscript 2) refer to the billet (steel in this case).

$C_1 = 4800$ ft/sec. $\quad \rho_1 = 62.4$ lb/ft^3

$C_2 = 1.8 \times 10^4$ ft/sec. $\quad \rho_2 = 500$ lb/ft^3

Hence,

$$\rho_1 C_1 + \rho_2 C_2 = (4.8 \times 6.24 \times 10^4 + 1.8 \times 5 \times 10^6) \frac{1}{32.2} \frac{\text{lb - sec}}{\text{ft}^3}$$

$$= (30 \times 10^4 + 9 \times 10^6) \frac{1}{32.2}$$

$$= 2.89 \times 10^5$$

Thus

$$u_2 = \frac{2 \times 215,000}{2.89 \times 10^5} = \frac{4.3}{2.89} = 1.49 \text{ ft/sec}$$

and

$$P_3 = \rho_2 u_2 C_2 = 2.8 \times 10^5 \times 1.49 = 416,000 \text{ psi}$$

These are the conditions immediately behind the wave front: for the unloading, $C_2 = u_2 = u_0$, and $P_3 = \Delta P$.

Hence, from equation (25) we have:

$$\frac{\Delta P}{8fy} = \ln \frac{r_0}{r} \text{ and with } fy = N = 90,000 \text{ psi}$$

$$\frac{\Delta P}{7.2\, 10^5} = \ln \frac{r_0}{r}$$. This equation is plotted in the following graph.

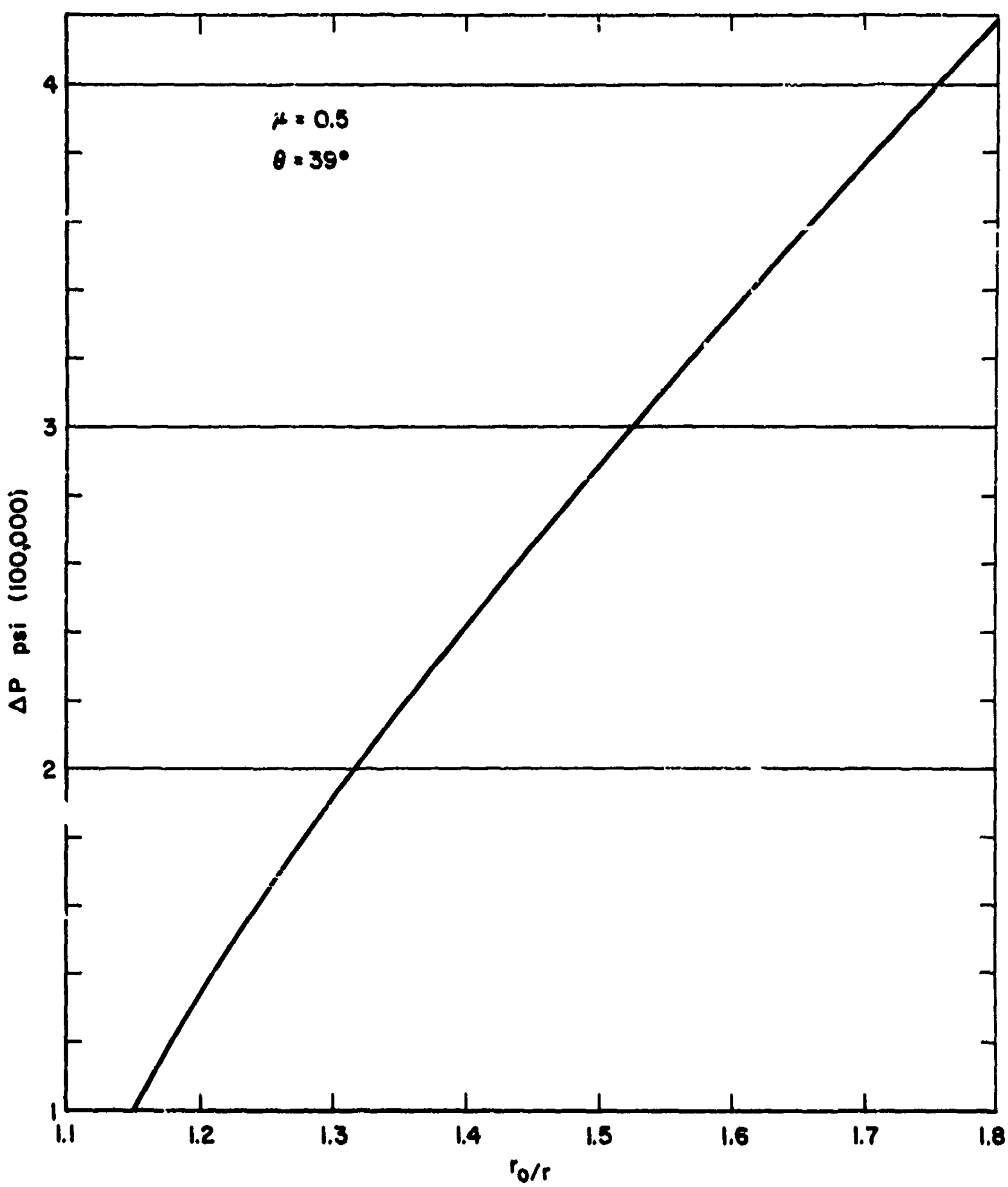


FIGURE 1

SECTION III - DISCUSSION

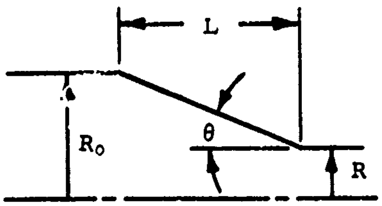
The result of the mathematical treatment is a simple equation similar to the one generally used for static extrusion. The two processes are not directly comparable, however.

In the static case, the pressure is raised until extrusion starts; this is the pressure necessary for extrusion for a given area reduction.

In the pulse extrusion case, on the other hand, the pulse propagates through the billet, causing the billet to move; while the extrusion for one pulse depends on the pressure amplitude, the final extrusion depends on the rate the pulses are generated.

The aforementioned plot of ΔP versus r_0/r, therefore, shows the extrusion for one pulse. The coefficient of friction was taken at 0.5 and the angle at 39° ($B = 0$ at $\theta = 39°$). A different choice of values for μ and θ can, of course, change the curve shown in the plot. To find the rate of extrusion, one must know the number of pulses per second.

For the extrusion of $\frac{R_0}{R} = \frac{8}{1}$ (i.e., $\frac{R_0}{R} = 8$ where R_0 is the radius of the billet before it enters the tapered section and R is the radius of the extruded section), $\mu = 0.5$, $\theta = 39°$ and $\Delta P = 400,000$ psi. The extrusion rate can be found as follows:



The number of pulses required to extrude the length L is given by:

$$\text{No. of pulses to extrude } L = \frac{R_0}{R} \Big/ \frac{r_0}{r}$$

The equivalent length of extruded stock for the length L is given by:

$$\pi R^2 \; l_1 = \frac{1}{3} \; \pi R_0{}^2 L \left[1 + \frac{R'}{R_0} + \left(\frac{R}{R_c}\right)^2\right]$$

Hence,

$$l_1 = \left(\frac{R_0}{R}\right)^2 L \left[1 + \frac{R}{R_0} + \left(\frac{R}{R_0}\right)^2\right]$$

$$= \frac{R_0}{\tan\theta} \left[\left(\frac{R_0}{R}\right)^2 + \frac{R_0}{R} + 1\right]$$

The total length extruded per second is therefore given by:

$$\frac{R_0}{\tan\theta} \left[\left(\frac{R_0}{R}\right)^2 + \frac{R_0}{R} + 1\right] \times \frac{\text{No. of pulses/sec}}{\text{No. of pulses to extrude } L}$$

For

$$\frac{R_0}{R} = 8, \qquad R_0 = 1'', \qquad \theta = 39°, \qquad \frac{r_0}{r} = 1.7 \quad \text{and}$$

$$\frac{R_0}{R} / \frac{r_0}{r} = \frac{8}{1.7} = 4.7 \text{ pulses to extrude } L'$$

The length extruded per second $= \frac{1}{.81} \times \left[64 + 8 + 1\right] \times \frac{(\text{No. of pulses/sec})}{4.7}$

$$= \frac{1 \times 73}{.81 \times 4.7} \times (\text{No. of pulses/sec})$$

$$= 19.2 \times (\text{No. of pulses/sec})$$

The length extruded per pulse, for $r_0 = 1$, is given by:

$$l_1 = \frac{r_0}{\tan\theta} \left[\left(\frac{r_0}{r}\right)^2 + \frac{r_0}{r} + 1\right]$$

$$= \frac{1}{.81} \; (2.9 + 1.7 + 1)$$

$$= \frac{5.6}{.81}$$

$$= 6.95 \text{ in./pulse}$$

The equivalent length of extruded stock for the length L is given by:

$$\pi R^2 \; l_1 = \frac{1}{3} \; \pi R_o^2 \, L \left[1 + \frac{R}{R_o} + \left(\frac{R}{R_o}\right)^2 \right]$$

Hence,

$$l_1 = \left(\frac{R_o}{R}\right)^2 L \left[1 + \frac{R}{R_o} + \left(\frac{R}{R_o}\right)^2 \right]$$

$$= \frac{R_o}{\tan\theta} \left[\left(\frac{R_o}{R}\right)^2 + \frac{R_o}{R} + 1 \right]$$

The total length extruded per second is therefore given by:

$$\frac{R_o}{\tan\theta} \left[\left(\frac{R_o}{R}\right)^2 + \frac{R_o}{R} + 1 \right] \times \frac{\text{No. of pulses/sec}}{\text{No. of pulses to extrude } L}$$

For

$$\frac{R_o}{R} = 8, \qquad R_o = 1'', \qquad \theta = 39°, \qquad \frac{r_o}{r} = 1.7 \quad \text{and}$$

$$\frac{R_o}{R} \Big/ \frac{r_o}{r} = \frac{8}{1.7} = 4.7 \text{ pulses to extrude } L$$

The length extruded per second $= \frac{1}{.81} \times \left[64 + 8 + 1 \right] \times \frac{(\text{No. of pulses/sec})}{4.7}$

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$$= 19.2 \times (\text{No. of pulses/sec})$$

The length extruded per pulse, for $r_o = 1$, is given by:

$$l_1 = \frac{r_o}{\tan\theta} \left[\left(\frac{r_o}{r}\right)^2 + \frac{r_o}{r} + 1 \right]$$

$$= \frac{1}{.81} \, (2.9 + 1.7 + 1)$$

$$= \frac{5.6}{.81}$$

$$= 6.95 \text{ in./pulse}$$

PART 2 - HIGH PRESSURE TRANSIENT LOADS IN A HOLLOW CYLINDER

The stresses and strains in thick-walled cylinders under internal loads can be calculated by Lamé's equations. When the internal load is transient, however, equilibrium of forces does not exist; Lamé's equation therefore are no longer applicable. Instead, the unbalanced forces will set up velocity gradients in part of the cylinder while some other part may be completely at rest.

Selberg[3] has given the solution to such problems for elastic loads. His results, however, are not explicitly available; it is not possible therefore to either modify his work to include plastic loads, or to use his results directly for plastic deformation.

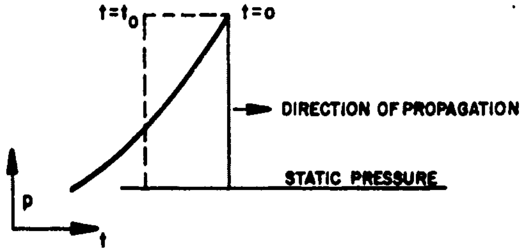
In this report the general problem is formulated and solutions are given explicitly for both the elastic and elasto-plastic cases. Rinehart[4] has carried out tests with thick-walled cylinders and high internal loads. His results indicate agreement with the prediction given in this report.

The solutions can be used for the analysis of stresses in shock tubes, gas guns, and pulse extrusion processes.

SECTION I - PROBLEM DESCRIPTION

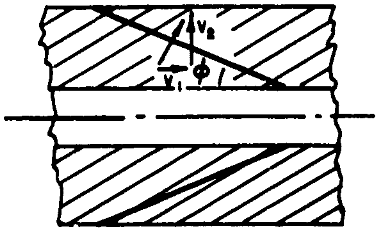
Consider a thick-walled cylinder of inside and outside radii "a" and "b," respectively. The cylinder is filled with gas or liquid to a static pressure P. Consider also a pressure disturbance, generated by either a fast accelerating piston or an electrical discharge, travelling from one end to the other: the disturbance is detonative with $\frac{d\sigma_1}{dt} = \infty$ at its front.

The pressure-time profile of the shock wave (detonative disturbance) is either rectangular or triangular.



In the triangular case, it is convenient to assume an equivalent rectangular wave with the areas $\int P\,dt$ and $\int P\,dx$ equal to the corresponding areas under the triangular profile. Thus, the momentum and energy carried by the wave are the same for either profile. The triangular wave can be treated separately, however, without difficulty.

As the wave travels along the cylinder, a stress wave travels radially. This wave, however, is not cylindrical. Its velocity of propagation has two components: v_1 the velocity of propagation in the medium contained by the cylinder and v_2, the velocity of propagation of the elastic wave in steel. It is assumed in this analysis that the angle ϕ is small (i.e., $v_2 \gg v_1$).

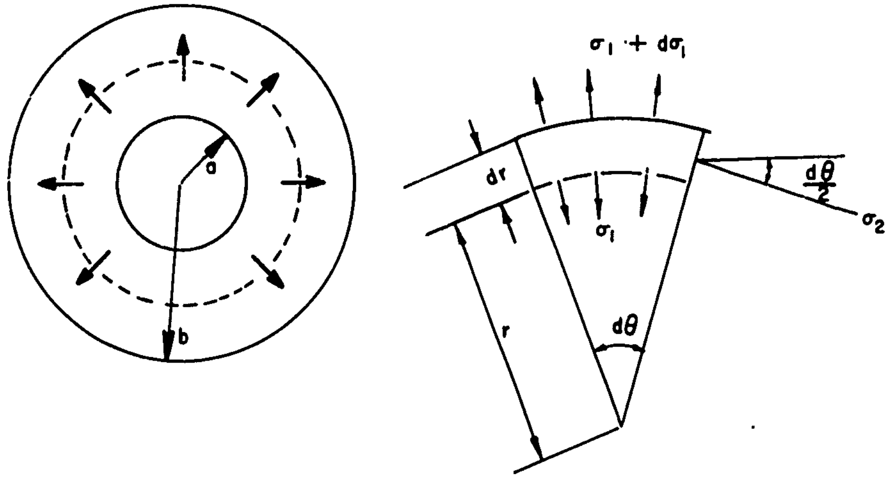


The problem, therefore, can be stated as follows: Given that a shock wave, whose magnitude and duration are known, travels along the inside bore of a thick-walled cylinder, what are the strain-time relations (both radial and hoop strain) in the wall of the cylinder if the yield stress of the material in the uniaxial test is known?

The resulting longitudinal wave is considered out of phase with the radial and therefore it is not considered in the analysis. It is, however, necessary to assume that the length L of the cylinder is considerably greater than the inside diameter of the cylinder.

SECTION II - PROBLEM FORMULATION

Consider an element at distance r from the center, of thickness dr and bounded by an angle $d\theta$.



Thus, when the wave arrives, the element is accelerated (or decelerated) according to Newton's third law,

$$- dm \cdot \frac{du}{dt} = \Sigma F$$

where ΣF = algebraic summation of forces

m = mass of the element

u = particle velocity

t = time

From Figure 1, it follows that:

$$+\uparrow \Sigma F = -\sigma_1 \cdot r\,d\theta + (\sigma_1 + d\sigma_1)(r + dr)\,d\theta - 2\sigma_2 \sin\frac{d\theta}{2} \cdot dr$$

but $\quad \frac{d\theta}{2} \cong \sin\frac{d\theta}{2}$

Hence:

$$\Sigma F = -\sigma_1 \cdot r \cdot d\theta + \sigma_1 \cdot r \cdot d\theta + \sigma_1 \cdot dr \cdot d\theta + d\sigma_1 \cdot r \cdot d\theta + dP \cdot dr \cdot d\theta - \sigma_2 \cdot d\theta \cdot dr$$

or $$\Sigma F = \sigma_1\, dr \cdot d\theta + d\sigma_1 \cdot r \cdot d\theta - \sigma_2 \cdot d\theta \cdot dr \qquad (1)^*$$

The mass of the element is given by:

$$dm = \left[\frac{r \cdot d\theta + (r + dr)\,d\theta}{2}\right] dr \cdot \rho$$

$$\doteq r \cdot d\theta \cdot dr \cdot \rho$$

where ρ is the mass density. Thus the momentum equation can be written as follows:

$$\rho\, r \cdot d\theta \cdot dr \cdot \frac{du}{dt} = \sigma_1 \cdot dr \cdot d\theta + d\sigma_1 \cdot r \cdot d\theta - \sigma_2 \cdot d\theta \cdot dr$$

or $$-\rho \frac{du}{dt} = \frac{\sigma_1}{r} + \frac{d\sigma_1}{dr} - \frac{\sigma_2}{r} \qquad (2)$$

but only a specified element has been considered so far: since $\frac{du}{dt}$ will change with the position of the particle, $\frac{du}{dt} = \frac{\partial u}{\partial t}$. Since $\frac{d\sigma_1}{dr}$ is for one instant of time only, $\frac{d\sigma_1}{dr} = \frac{\partial \sigma_1}{\partial r}$. Hence, equation (2) can be written as follows:

$$-\rho \frac{\partial u}{\partial t} = \frac{\sigma_1}{r} + \frac{\partial \sigma_1}{\partial r} - \frac{\sigma_2}{r} \qquad (3)$$

The continuity equation is derived from the fact that the mass of the element will not change with the passing of the wave. Hence, an element initially has

---

* The equations in Part II are numbered independently of Part I.

density ρ_0 and occupies a length dr_0: during the passing of the wave, the density changes to the value ρ and the element now occupies a length dr. Thus

$$dh = \rho_0 \cdot r_0 \cdot d\theta \cdot dr_0 = \rho \cdot r \cdot d\theta \cdot dr = \text{mass per unit radius}$$

and $$\frac{\partial r}{\partial r_0} = \frac{\rho_0 r_0}{\rho \cdot r} \tag{4}$$

Here again, the total derivatives were considered for one element at one instant of time, hence they are now expressed in partial derivatives. The problem can be better formulated by using the Lagrangian representation of $r = r(r_0, t)$ where r_0 is the initial distance of the center of gravity of the element from the center: thus, r_0 and t are chosen as the independent variables.

Equation (4) can be rewritten as follows:

$$\frac{\partial}{\partial r} = \frac{\partial}{\partial r_0} \cdot \frac{\partial r_0}{\partial r} = \frac{\rho r}{\rho_0 r_0} \frac{\partial}{\partial r_0} \tag{5}$$

Substituting (5) into (3), we get

$$-\rho \frac{\partial u}{\partial t} = \frac{\sigma_1 - \sigma_2}{r} + \frac{\rho \cdot r}{\rho_0 \cdot r_0} \frac{\partial \sigma_1}{\partial r_0}$$

Hence,

$$\frac{\partial u}{\partial t} + \frac{r}{\rho_0 \cdot r_0} \frac{\partial \sigma_1}{\partial r_0} + \frac{\sigma_1 - \sigma_2}{\rho \cdot r} = 0 \tag{6}$$

Equation (4) is now differentiated with respect to time, t, and hence,

$$\frac{\partial}{\partial t} (\rho \cdot r \cdot \frac{\partial r}{\partial r_0}) = 0$$

or $$\rho \cdot r \cdot \frac{\partial u}{\partial r_0} + r \cdot \frac{\partial r}{\partial r_0} \frac{\partial \rho}{\partial t} + \rho \cdot \frac{\partial r}{\partial r_0} u = 0 \tag{7}$$

where $u = \frac{\partial r}{\partial t}$ (by definition)

and $$\rho \cdot r \cdot \frac{\partial u}{\partial r} + r \frac{\partial \rho}{\partial t} + \rho \cdot u = 0 \tag{8}$$

Now consider the radial and hoop strains:

$$\text{Radial strain} \quad \epsilon_1 = \frac{\frac{\partial r}{\partial r_o} dr_o - dr_o}{dr_o} = \frac{\partial r}{\partial r_o} - 1$$

$$\text{Hoop strain} \quad \epsilon_2 = \frac{2\pi r - 2\pi r_o}{2\pi r_o} = \frac{r}{r_o} - 1$$

Hence $\qquad \epsilon_1 + 1 = \dfrac{\partial r}{\partial r_o} \qquad (9)$

and $\qquad \epsilon_2 + 1 = \dfrac{r}{r_o} \qquad (10)$

Combining (4) and (9), we obtain

$$(\epsilon_1 + 1) \cdot \rho \cdot r = \rho_o \cdot r_o \qquad (11)$$

Differentiating (11) with respect to t, gives

$$\frac{\partial}{\partial t} \left\{ (\epsilon_1 + 1) \cdot \rho \cdot r \right\} = 0$$

$$(\epsilon_1 + 1) \rho \cdot \frac{\partial r}{\partial t} + (\epsilon_1 + 1) r \cdot \frac{\partial \rho}{\partial t} + \rho \cdot r \cdot \frac{\partial \epsilon_1}{\partial t} = 0$$

and $\qquad r \cdot \dfrac{\partial \rho}{\partial t} = - \rho \cdot u - \dfrac{\rho \cdot r}{(\epsilon_1 + 1)} \dfrac{\partial \epsilon_1}{\partial t}$

$\therefore \qquad \rho \cdot r \cdot \dfrac{\partial u}{\partial r} - \dfrac{\rho \cdot r}{(\epsilon_1 + 1)} \dfrac{\partial \epsilon_1}{\partial t} = 0$

Using equation (5),

$$\frac{(\rho \cdot r)^2}{\rho_o \cdot r_o} \frac{\partial u}{\partial r_o} - \frac{\rho \cdot r}{\epsilon_1 + 1} \frac{\partial \epsilon_1}{\partial t} = 0$$

and $\qquad \dfrac{\partial u}{\partial r_o} - \dfrac{\partial \epsilon_1}{\partial t} = 0 \qquad (12)$

The problem is thus defined with two partial differential equations:

$$\frac{\partial u}{\partial t} + \frac{r}{\rho_o \cdot r_o} \frac{\partial \sigma_1}{\partial r_o} + \frac{\sigma_1 - \sigma_2}{\rho \cdot r} = 0 \tag{6}$$

and

$$\frac{\partial u}{\partial r_o} - \frac{\partial \epsilon_1}{\partial t} = 0 \tag{12}$$

For the solution of this system, r_o and t are the independent variables and σ_1 and u are the dependent variables. Furthermore, $\rho = \rho\,(\rho_o, \sigma_1)$, $r = r\,(r_o, t)$, and $\sigma_2 = \sigma_2\,(r_o, \sigma_1)$: thus the coefficients of (6) and (12) are functions of a combination of u, σ_1, r_o, and t. Equation (12) has, however, ϵ_1 as one of the dependent variables. This indicates that an additional equation is required to solve equations (6) and (12) simultaneously. The equation will be derived from the equation of state of the material. For metals, an equation of stress-strain will be sufficient.

For a biaxial stress field it is known that

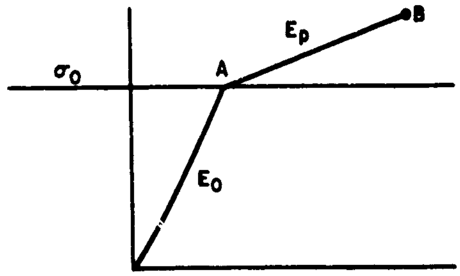
$$\epsilon_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} \tag{13}$$

(N.B.: The cylinder is not restrained in the axial direction.)

Whenever the stresses extend into the plastic region, the problem must be divided into two parts. The first part takes place in the elastic region as defined by Tresca's criterion of failure, i.e.,

$$\left| \frac{\sigma_2 - \sigma_1}{2} \right| = \frac{\sigma_y}{2}$$

assuming that the stresses σ_2 and σ_1 are of opposite sign. The second part takes place in the plastic region. An elastic-strain hardening bilinear relation for the plastic region can be assumed as follows:



Hence

$$(\sigma - \sigma_o) = E_p\,(\epsilon - \epsilon_o) \tag{14}$$

and

$$\epsilon_{1p} = \frac{\sigma_{1p}}{E_p} - \mu_p\,\frac{\sigma_{2p}}{E_p} \tag{15}$$

In general we can write

$$\frac{\partial \epsilon_1}{\partial t} = \frac{1}{E}\,\frac{\partial \sigma_1}{\partial t} - \frac{\mu}{E}\,\frac{\partial \sigma_2}{\partial t} \tag{16}$$

But $\quad \dfrac{\sigma_2}{E} = \dfrac{r}{r_o} - 1$

$$\therefore \qquad \frac{1}{E}\,\frac{\partial \sigma_2}{\partial t} = \frac{1}{r_o}\,\frac{\partial r}{\partial t} = \frac{u}{r_o} \tag{17}$$

Hence

$$\frac{\partial \epsilon_1}{\partial t} = \frac{1}{E} - \frac{\mu \cdot u}{r_o} \tag{18}$$

Equation (18) is now substituted in equation (12), giving

$$\frac{\partial u}{\partial r_o} - \frac{1}{E}\,\frac{\partial \sigma_1}{\partial t} + \frac{\mu}{r_o}\,u = 0$$

Rewriting equation (6) we also have

$$\frac{\partial u}{\partial t} \mp \frac{r}{\rho_o \cdot r_o}\,\frac{\partial \sigma_1}{\partial r_o} \mp \frac{\sigma_1 - \sigma_2}{\rho \cdot r} = 0$$

(19)

The solution of equations (19) can be afforded by the theory of characteristics.

The general equations are as follows:

$$A_1\left(\frac{\partial U}{\partial X}\right)_Y + B_1\left(\frac{\partial U}{\partial Y}\right)_X + C_1\left(\frac{\partial V}{\partial X}\right)_Y + D_1\left(\frac{\partial V}{\partial X}\right)_Y + E_1 = 0$$

$$A_2\left(\frac{\partial U}{\partial X}\right)_Y + B_2\left(\frac{\partial U}{\partial Y}\right)_X + C_2\left(\frac{\partial V}{\partial X}\right)_Y + D_2\left(\frac{\partial V}{\partial Y}\right)_X + E_2 = 0$$

where the coefficients A_1, B_1, C_1, D_1, E_1, A_2, B_2, C_2, D_2, E_2 are functions of U, V, X and Y. Now let $u = U$, $\sigma_1 = V$, $X = r_o$ and $Y = t$. Thus, rewriting equations (19), we have

$$\left(\frac{\partial u}{\partial r_o}\right)_t + 0\left(\frac{\partial u}{\partial t}\right)_{r_o} + 0\left(\frac{\partial \sigma_1}{\partial r_o}\right)_t - \frac{(1)}{E}\left(\frac{\partial \sigma_1}{\partial t}\right)_{r_o} + \frac{\mu}{r_o} u = 0$$

$$0\left(\frac{\partial u}{\partial r_o}\right)_t + \left(\frac{\partial u}{\partial t}\right)_{r_o} + \frac{r}{\rho_o \cdot r_o}\left(\frac{\partial \sigma_1}{\partial r_o}\right)_t + 0\left(\frac{\partial \sigma_1}{dt}\right)_{r_o} + \frac{\sigma_1 - \sigma_2}{\rho \cdot r} = 0$$

The coefficients, therefore, are as follows:

$A_1 = 1$	$A_2 = 0$
$B_1 = 0$	$B_2 = 1$
$C_1 = 0$	$C_2 = -\frac{r}{\rho_o r_o}$
$D_1 = -\frac{1}{E}$	$D_2 = 0$
$E_1 = \frac{\mu u}{r_o}$	$E_2 = -\frac{1}{\rho r}(\sigma_1 - \sigma_2)$

The solution is now given.

$$\frac{dr_o}{dt} = \pm\sqrt{\frac{E}{\rho_o}\,\frac{r}{r_o}} \tag{20}$$

and $$du + \Phi \frac{r}{\rho_0 \cdot r_0} \, d\sigma_1 + \left[\frac{\mu u}{r_0} + \frac{\sigma_1 - \sigma_2}{\rho \cdot r} \Phi \right] dr_0 = 0 \tag{21}$$

where $$\Phi = \frac{dt}{dr_0} \ . \tag{22}$$

The method of characteristics has thus replaced the two partial differential equations with two total differential equations. Equation (20) has the units of velocity and, in fact, is the propagation velocity in the Lagrangian designation. Equation (21) shows the condition of the metal behind the propagating front.

SECTION III - BOUNDARY CONDITION

To find the boundary condition, or the motion of the interface, at time $t = 0$, consider an element dr as before.(See Figure 1).

Hence $\quad dm \frac{du}{dt} = \Delta F$

and $\quad dm = \rho \cdot r \cdot d\theta \cdot dr = \rho r_o d\theta dr_o$

$$\uparrow + \text{ve } \Delta F = - \sigma_1 \cdot d\theta \cdot r + (\sigma_1 + d\sigma_1) \cdot (\zeta + dr) \cdot d\theta - 2\sigma_2 \sin \frac{d\theta}{2} dr$$

$$= + d\sigma_1 \cdot r \cdot d\theta + \sigma_1 \cdot dr \cdot d\theta - \sigma_2 \cdot d\theta \cdot dr$$

Therefore $\quad \rho \cdot r \cdot dr \frac{du}{dt} = d\sigma_1 \cdot r + \sigma_1 \cdot dr - \sigma_2 \cdot dr$

or $\quad \rho \cdot r \frac{dr}{dt} du = d\sigma_1 \cdot r + (\sigma_1 - \sigma_2) dr$

since $\quad r = r(r_o, t)$

Thus $\quad \frac{dr}{dt} = u + (1+\epsilon) C \left(\text{from } \frac{dr}{dt} = \frac{\partial r}{\partial t} + \frac{\partial r}{\partial r_o} \frac{dr_o}{dt} = u + (1+\epsilon) C\right)$

or $\quad \rho \cdot r u du + \rho \cdot r (1+\epsilon) C du = d\sigma_1 \cdot r + (\sigma_1 - \sigma_2) dr$

Now let $\quad dr \to 0$, i.e., $r \to r_o$ $(dr = d(r - r_o))$

Hence $\quad \epsilon \to 0$ and $u \to u_o$

$$\rho \cdot u_o du_o + \rho \cdot C_o du_o = d\sigma_1$$

$$\rho(u_o + C_o) du = d\sigma_1$$

but $\quad u_o + C_o = w_o$ (propagation velocity at $t = t_o$)

$$\therefore \qquad \rho\, w_o du_o = d\sigma_1$$

and for a sudden change,

$$\underline{\rho \cdot w_1 \cdot u_1 = \rho} \qquad (23)$$

SECTION IV - ENERGY CONSIDERATION

The conservation of energy in the cylindrical stress wave front can be written as follows:

$$u_1^2 \rho_1 \pi r_1 dr = \rho_2 \pi r_2 \cdot dr u_2^2 + E_{losses}$$

It can be assumed that E_{losses} (strain energy losses) is negligible for moderate strains. Thus it can be stated that for all stress wave cases where excessive plastic "stretching" does not occur, the kinetic energy per unit radius remains constant. Hence

$$u_1^2 r_1 = u_2^2 r_2 \quad \left(\text{i.e., } \frac{dE}{dr} = \text{constant}\right)$$

or

$$u_2 = u_1 \sqrt{\frac{r_1}{r_2}} \tag{24}$$

(The density in this context is assumed constant.)

It is thus possible to find the velocity at any radius once the velocity of the bare surface (boundary) is known. From equation (23) u_1 is found as follows:

$w_1 \simeq C_1$ (the acoustic speed) since $C_1 \gg u_1$

P = internal pressure.

SECTION V - SOLUTION OF EQUATION 21

A. GENERAL SOLUTION

It was found previously in equation (20) that:

$$C = \frac{1}{\Phi} = \frac{dr_o}{dt} = \pm\sqrt{\frac{E}{\rho_o}\ \frac{r}{r_o}}$$

and

$$du + \Phi\,\frac{r}{\rho_o r_o}\cdot\ d\sigma_1 + \left[\mu\,\frac{u}{r_o} + \frac{\sigma_1 - \sigma_2}{\rho\cdot r}\,\Phi\right]dr_o = 0 \qquad (21)$$

These equations can be rewritten as follows:

$$\frac{du}{dt} \pm \frac{1}{E}\sqrt{\frac{E}{\rho_o}\cdot\frac{r}{r_o}}\cdot\frac{d\sigma_1}{dt} + \mu\,\frac{u}{r_o}\cdot\frac{dr_o}{dt} + \frac{(\sigma_1 - \sigma_2)}{\rho_o\cdot r_o}(\epsilon + 1) = 0$$

or

$$\frac{du}{dt} \pm \frac{C}{E}\ \frac{d\sigma_1}{dt} + \frac{\mu\cdot u\cdot C}{r_o} + \frac{(\sigma_1 - \sigma_2)}{\rho_o r_o}(\epsilon + 1) = 0$$

Before any attempt to integrate the above equation is made, it is important to list the events in sequence.

- At $t = 0$, the internal pressure P is applied suddenly.

From equation (23), u_1 is found. The velocity and density changes immediately behind the front are sudden and take place almost instantaneously upon arrival of the shock front. This is a characteristic of shock waves generally. On the other hand, the hoop and axial strains will only take place with time, as the individual particles start moving under velocity u. Thus, from equation (11) $\left(\epsilon_1 = \frac{\rho_o r_o}{\rho\cdot r} - 1\right)$ it is easily seen that while $\frac{r}{r_o} = 1$ (at t=0), the radial strain is only a function of density changes $\left(\epsilon_1 = \frac{\rho_o}{\rho} - 1\right)$. Furthermore, the sudden

radial change will be restrained both in the hoop and axial directions.

Hence, at $t = 0$,

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu^2 \frac{\sigma_1}{E} - \mu^2 \frac{\sigma_1}{E}$$

The elastic loading will thus be given by

$$\frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_1 - \mu\sigma_1}{2}$$

or $\qquad \sigma_1 (1-\mu) = \sigma_y \qquad (25)$

(i.e., $\epsilon_3 = \epsilon_2 = -\mu \frac{\sigma_1}{E}$. Since this strain is restrained, it affects the radial strain as $\mu \left(\mu \frac{\sigma_1}{E}\right)$)

and $\qquad \epsilon_1 = \frac{\sigma_1}{E} (1 - 2\mu^2) \qquad (26)$

Thus, $\qquad \frac{\rho_o}{\rho} = 1 + \frac{\sigma_1}{E} (1 - 2\mu^2) \qquad (27)$

at $t = 0$ and $\sigma_1 = \frac{\sigma_y}{1-\mu}$. If $\sigma_1 > \frac{\sigma_y}{1-\mu}$, ζ is given by the following equation.*

$$\frac{\rho_o}{\rho} = \zeta = \frac{C_o - u}{C_o} \qquad (28)$$

---

* The continuity equation across the wave front can be written for $t = 0$ as follows:

$$\rho (C_o - u) = \rho_o C_o.$$

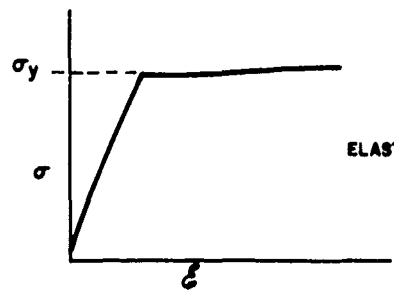
Hence $\qquad \frac{\rho_o}{\rho} = \zeta = \frac{C_o - u}{C_o}$

Thus, if C_o and u behind the wave are known, ζ can be calculated.

For E = 0, C = 0 and hence, $\frac{\rho}{\rho_0} = 0$ or $\frac{\rho_0 + \Delta\rho}{\rho_0} = 0$. This indicates that $\frac{\rho}{\rho_0}$ does not change for $t > 0$ if E = 0.

For $0 < t < t_0$, P remains constant; thus $\frac{d\sigma_1}{dt} = 0$ (for triangular waves, $\frac{d\sigma_1}{dt} =$ constant).

For one particular case, the model of the stress-strain curve is assumed as follows:



Immediately behind the loading wave ($\bar{t} = 0$), however, a triaxial stress field exists and the maximum radial strain that can be applied elastically is

$$\sigma_1 = \frac{\sigma_y}{1-\mu} \tag{29}$$

The axial and hoop stresses are compressive at t = 0. For $\sigma_1 > \frac{\sigma_y}{1-\mu}$, the ratio $\frac{\rho_0}{\rho}$ is constant, i.e., plastic deformation takes place at constant density. At $\bar{t} = 0$, $\frac{r}{r_0} = 1$. Equation (29) shows that the elastic limit at $\bar{t} = 0$ and $\frac{r}{r_0} = 1$ is higher than for a static case wherein the application of σ_1 is associated with corresponding hoop stress (of opposite sign). Thus the plastic deformation begins at $\sigma_1 - \sigma_2 = \sigma_y$.

For a compressive internal pressure, σ_1 is negative and σ_2 is positive. Thus

$$- |\sigma_1| - |\sigma_2| = |\sigma_y| < \left(\frac{|\sigma_y|}{1-\mu}\right)_{shock} \tag{30}$$

With these considerations in mind, equation 21 can be written

$$\frac{du}{dt} + \frac{\mu \cdot u \cdot C}{r_o} + \frac{(\sigma_1 - \sigma_2)}{\rho_o \cdot r_o} \zeta \left(\frac{r_o}{r}\right) = 0 \tag{31}$$

It is convenient to express the different variables in dimensionless form as follows:

$$\bar{u} = \frac{u}{C_o} \qquad \bar{t} = \frac{t \cdot C_o}{r_o} \qquad \bar{C} = \frac{C}{C_o}$$

$$\bar{\sigma}_1 = \frac{\sigma_1}{E_o} \qquad \bar{E} = \frac{E}{E_o} \qquad C_o = \sqrt{\frac{E_o}{\rho_o}}$$

Hence, by substituting in (31),

$$\frac{d\bar{u}}{d\bar{t}} + \mu \cdot \bar{u} \cdot \bar{C} + (\bar{\sigma}_1 - \bar{\sigma}_2) \zeta \left(\frac{r_o}{r}\right) = 0 \tag{32}$$

$$C = \sqrt{\frac{E}{\rho_o} \frac{r}{r_o}} = \sqrt{\frac{E}{E_o} \frac{E_o}{\rho_o} \frac{r}{r_o}} = C_o \sqrt{\bar{E} \frac{r}{r_o}}$$

Since $\quad r = f(r_o, t)$

$$\frac{dr}{dt} = \left(\frac{\partial r}{\partial t}\right) + \left(\frac{\partial r}{\partial r_o}\right) \frac{dr_o}{dt} = u + \zeta \frac{r_o}{r} C$$

and

$$\frac{d\left(\frac{r}{r_o}\right)}{d\bar{t}} = \bar{u} + \zeta \frac{r_o}{r} \bar{C} \tag{33}$$

Now, consider the case where $\sigma_1 = \frac{\sigma_y}{1-\mu}$.

The "stretching out" at $t>o$ takes place in the plastic region. Since $\bar{C} = C = 0$ in this region, equations (32) and (33) can be rewritten as follows:

$$\frac{d\bar{u}}{d\bar{t}} + (\bar{\sigma}_1 - \bar{\sigma}_2)\,\zeta \left(\frac{r_o}{r}\right) = 0$$

$$\frac{d\left(\frac{r}{r_o}\right)}{d\bar{t}} = \bar{u} \qquad (34)$$

B. LOADING WAVE SOLUTION

Let $\frac{r}{r_o} = 1 + \xi$

where $\xi \ll 1$

$$\frac{d\xi}{d\bar{t}} = u$$

Hence, $\frac{d^2\xi}{d\bar{t}^2} = \frac{d\bar{u}}{d\bar{t}}$

Therefore, $\frac{d^2\xi}{d\bar{t}^2} + \frac{\bar{\sigma}_y \zeta}{(1+\xi)} = 0$

where, from Tresca's criterion, $\bar{\sigma}_y = (\bar{\sigma}_1 - \bar{\sigma}_2)$.

$$\frac{d^2\xi}{d\bar{t}^2} - \bar{\sigma}_y\,\zeta\xi = -\,\bar{\sigma}_y\,\zeta\,. \qquad (35)$$

The quantity $\bar{\sigma}_y$ is negative since,we are considering compression. Thus (35) can be rewritten

$$\frac{d^2\xi}{d\bar{t}^2} + \bar{\sigma}_y\,\zeta\xi = \bar{\sigma}_y\,\zeta \qquad (36)$$

The solution of this equation is:

$$\xi = C_1\,e^{\omega t} + C_2 e^{-\omega t} - 1 \qquad (37)$$

where $\qquad \omega = \sqrt{\bar{\sigma}_y \zeta}$ $\qquad$ (38)

The conditions at $\bar{t} = 0$ are $\ell = 0$ and $\frac{d\ell}{dt} = \bar{u}_o$. Hence,

$$\left.\begin{aligned} C_1 + C_2 &= 1 \\ C_1 - C_2 &= \frac{\bar{u}_o}{w} \end{aligned}\right\} \text{ thus } \quad \begin{aligned} C_1 &= (1 + \frac{u_o}{w}) \frac{1}{2} \\ C_2 &= \frac{1}{2}(1 - \frac{u_o}{w}) \end{aligned}$$

Substituting in (37) gives

$$\ell = \left(1 + \frac{u_o}{w}\right)\frac{1}{2} e^{\omega\bar{t}} + \left(1 - \frac{u_o}{w}\right)\frac{1}{2} e^{-\omega t} - 1$$

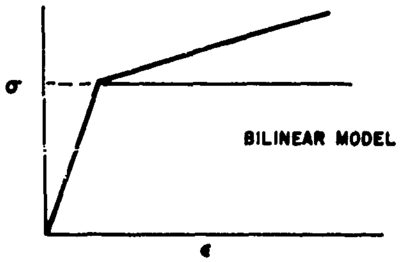
$$= \frac{e^{\omega\bar{t}} + e^{-\omega t}}{2} + \frac{u_o}{w}\left(\frac{e^{\omega\bar{t}} - e^{-\omega\bar{t}}}{2}\right) - 1$$

and

$$\ell = \cosh \omega\bar{t} + \frac{\bar{u}_o}{w} \sinh \omega\bar{t} - 1 \qquad (39)$$

Equation (39) shows how the hoop strain changes with $\bar{t}$ for a given initial velocity $\bar{u}_o$ and parameter ω.

It is also possible to find a solution for the elastic-strain hardening model.



In this case, $C_{plastic}$ is not zero: instead, it has a value given by

$$C = C_o \sqrt{\bar{E} \frac{r}{r_o}} \qquad (40)$$

where $\bar{E} = \frac{E_{plastic}}{E_{elastic}}$. Equation (33) can therefore be rewritten as follows:

$$\frac{d\left(\frac{r}{r_o}\right)}{d\bar{t}} = \bar{u} + \zeta \frac{r_o}{r} \cdot \bar{C} = \bar{u} + \zeta \frac{\bar{E}}{\bar{C}} = \bar{u} + \zeta \frac{\bar{E}^{1/2}}{\left(\frac{r}{r_o}\right)^{1/2}} \tag{41}$$

Let $\frac{r}{r_o} = 1 + \mathcal{E}$, where $\mathcal{E} \ll 1$. Hence

$$\frac{d\mathcal{E}}{d\bar{t}} = \bar{u} + \zeta \bar{E}^{1/2} (1 - 1/2\, \mathcal{E})$$

Thus $$\frac{d^2\mathcal{E}}{d\bar{t}^2} = \frac{d\bar{u}}{d\bar{t}} - 1/2\, \zeta\, \bar{E}^{1/2} \frac{d\mathcal{E}}{dt} \tag{42}$$

Equation (32) can also be rewritten as follows:

$$\frac{d^2\mathcal{E}}{d\bar{t}} + 1/2\, \zeta\, \bar{E}^{1/2} \frac{dr}{dt} + \mu \bar{E}^{1/2} (1 + 1/2\, \mathcal{E})\left(\frac{d\mathcal{E}}{d\bar{t}} + 1/2\, \zeta \bar{E}^{1/2} \mathcal{E} - \zeta \bar{E}^{1/2}\right) + \bar{F} \zeta (1 - \mathcal{E}) = 0 \tag{43}$$

Rearranging equation (43), we have

$$\frac{d^2\mathcal{E}}{d\bar{t}} + \left(1/2\, \zeta + \mu\right) \bar{E}^{1/2} \frac{d\mathcal{E}}{d\bar{t}} - \bar{\sigma}_y \zeta \mathcal{E} = \zeta(\mu \bar{E} - \bar{\sigma}_y) \tag{44}$$

Equation (44) has the following general solution:

$$\mathcal{E} = C_1 e^{-\frac{\alpha}{2} + \frac{1}{2}\sqrt{\alpha^2 + 4\bar{\sigma}_y \zeta}} + C_2 e^{-\frac{\alpha}{2} - \frac{1}{2}\sqrt{\alpha^2 + 4\bar{\sigma}_y \zeta}} - \left(\frac{\mu \bar{E} - \bar{\sigma}_y}{\bar{\sigma}_y}\right) \tag{45}$$

where $\alpha = (\frac{1}{2}\zeta + \mu)\, \bar{E}^{1/2}$. The quantity under the radical determines the nature of loading. There are three possibilities:

(1) $(\frac{1}{2}\zeta + \mu)^2\, \bar{E} > 4\bar{\sigma}_y \zeta$, i.e., $\bar{E} > \frac{4\bar{\sigma}_y \zeta}{(\frac{1}{2}\zeta + \mu)^2}$

In this case, the ensuing motion is decaying without oscillations.

(2) $0 < \bar{E} < \dfrac{4\bar{\alpha}_\gamma \zeta}{(\frac{1}{2}\zeta+\mu)^2}$; the motion is decaying with oscillations.

(3) $\bar{E} = 0$. This case was shown previously: it is decaying without oscillation.

The initial conditions for the solution of equation (45) are:

$$\left.\begin{aligned} &\mathcal{E} = 0 \text{ at } \bar{t} = 0 \\ \text{and} \quad &\left(\frac{d\mathcal{E}}{dt}\right)_{t=0} = \bar{u}_o + \zeta \bar{E}^{1/2} \end{aligned}\right\} \qquad (46)$$

where $\bar{u}_o$ is given by equation (23).

Thus we have the following final expressions:

(1) $\bar{E} > \dfrac{4\bar{F}\zeta}{(\frac{1}{2}\zeta+\mu)^2}$

$$\mathcal{E} = \left[\frac{\gamma}{\beta}\cosh\frac{\psi}{2}\bar{t} + Y\sinh\frac{\psi}{2}\bar{t}\right]e^{-\frac{\alpha}{2}\bar{t}} \quad \frac{\gamma}{\beta} \qquad (47)$$

and

$$\bar{u} = \left\{\frac{\gamma}{\beta}\frac{\psi}{2} - \frac{E\alpha}{2} + \frac{1}{2}\zeta(\bar{E})^{1/2}Y\right\}\sinh\frac{\psi}{2}\bar{t}$$

$$+\left\{Y\frac{\psi}{2} - \frac{\alpha}{2}\frac{\gamma}{\beta} + \frac{1}{2}\zeta(\bar{E})^{1/2}\frac{\gamma}{\beta}\right\}\cosh\frac{\psi}{2}\bar{t}$$

$$-\zeta(\bar{E})^{1/2}\left(\frac{1}{2}\frac{\gamma}{\beta} + 1\right)e^{\frac{\alpha}{2}\bar{t}} \qquad (48)$$

(2) $0 < \bar{E} < \dfrac{4\bar{F}\zeta}{(\frac{1}{2}\zeta+\mu)^2}$

$$\mathcal{E} = Ae^{-\frac{\alpha}{2}\bar{t}}\cos\left(\frac{\psi}{2}\bar{t} - \delta\right) - \frac{\gamma}{\beta}$$

and $$\bar{u} = A\left[\left(\frac{1}{2}\zeta\bar{E}^{1/2} - \frac{\alpha}{2}\right)\cos\left(\frac{\psi}{2}\bar{t} - \delta\right) - \frac{\psi}{2}\sin\left(\frac{\psi}{2}\bar{t} - \delta\right)\right]e^{-\frac{\alpha}{2}\bar{t}}$$

$$- \zeta\bar{E}^{1/2}\left(\frac{1}{2}\frac{\gamma}{\beta} + 1\right)$$

where

$$\psi = \sqrt{\alpha^2 + 4\bar{\sigma}_y\zeta} \qquad \alpha = \left(\frac{1}{2}\zeta + \mu\right)\bar{E}^{1/2}$$

$$\gamma = \zeta(\mu\bar{E} - \bar{\sigma}_y) \qquad Y = \frac{\xi + \frac{\alpha}{2}\frac{\gamma}{\beta}}{\frac{\psi}{2}}$$

$$\beta = \bar{\sigma}_y\zeta \qquad \xi = u_o + \frac{1}{2}\zeta\bar{E}^{1/2}$$

and the integration constants A and δ are given by

$$A = \frac{\gamma}{\beta} \cdot \frac{1}{\cos\sigma} \tag{49}$$

and $$\tan\delta = \frac{\beta Y}{\gamma} \tag{50}$$

C. UNLOADING WAVE SOLUTION

The unloading wave is fully elastic, unless the conditions are such that secondary yielding in the opposite direction occurs. The assumption is made that, immediately after the sudden unloading, the material recovers its volume and thus $\frac{\rho_o}{\rho} = \zeta = 1$. The procedure of formulation and solution is the same as for the loading. Now, however, $\bar{E} = 1$ and $\frac{r}{r_o} = 1 - \mathcal{S}$.

The solution is given by the following equations:

$$\mathcal{S} = e^{-\frac{\alpha}{2}\bar{t}}\left[\frac{\gamma}{\beta}\cosh\frac{\psi}{2}\bar{t} + Y\sinh\frac{\psi}{2}\bar{t}\right] - \frac{\gamma}{\beta} \tag{51}$$

where, for Y, $\xi = -1 - u_o$.

$$\bar{u} = \left[\frac{\gamma}{\beta}\frac{\psi}{2} - \frac{\alpha}{2} Y + \frac{1}{2} Y\right] \sinh \frac{\psi}{2}\bar{t}$$

$$+ \left\{\frac{\psi}{2} Y - \frac{\alpha}{2}\frac{\gamma}{\beta} + \frac{1}{2}\frac{\gamma}{\beta}\right\} \cos \frac{\psi}{2}\bar{t} - \left[\frac{1}{2}\frac{\gamma}{\beta} - 1\right] e^{\frac{\alpha}{2}\bar{t}} \tag{52}$$

where

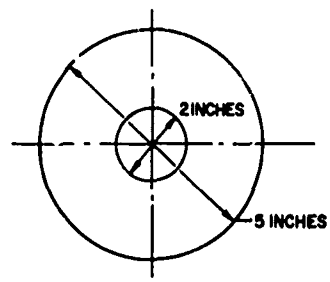
$$\left.\begin{aligned} \psi &= \sqrt{\alpha^2 + 4\bar{\sigma}_y} \\ \alpha &= \frac{1}{2} + \mu \\ \beta &= 1 - \mu + \bar{\sigma}_1 \\ \gamma &= \mu - \bar{\sigma}_1 \end{aligned}\right\} \tag{53}$$

D. CALCULATING PROCEDURE

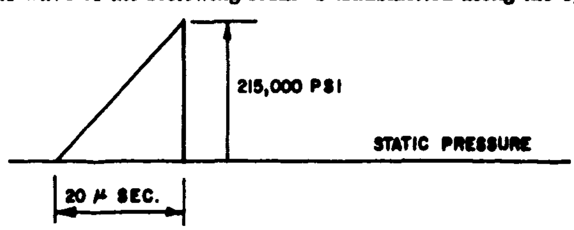
The velocity equation is used in both the loading and unloading cases, to find the decay of the velocity and, finally, to find the time that it becomes zero. This time is then substituted in the strain equation to find the resulting strain.

SECTION VI - EXAMPLE

Consider a thick-walled cylinder with σ_y = 250,000 psi and the following dimensions (the cylinder is filled with water):



A triangular wave of the following form is transmitted along the cylinder.



This wave reflects upon arrival on a closed end. Find the resulting strains r_o = 1 inch and r_o = 3 inches.

The pressure of the reflected wave front is approximately twice the pressure of the incident wave front. This is considered the highest possible internal pressure; thus, the calculations will be performed for the reflected region only.

To illustrate the wave decay, two cases are analyzed; Case 1, where $r_o = 1$ inch, and Case 2, where $r_o = 3$ inches. Hence $P = -2\,(215,000) = -430,000$ psi at $r_o = 1$ inch. Using equation (23), we have

$$u_1 = - \frac{430,000\,g}{\rho_o\,C_o} = - 2864.6 \text{ in./sec}$$

where $\rho_o = 0.29 \text{ lb/in.}^3$

$C_o = 200,000 \text{ in./sec}$

$g = 32.2 \text{ ft/sec}^2$

From equation (34), the velocity at $r_o = 3$ inches (chosen arbitrarily) is

$$u_3 = u_1 \sqrt{\frac{r_1}{r_3}} = - 2,864.6 \sqrt{\frac{1}{3}} = - 1654.0 \text{ in./sec}$$

Thus the pressure at $r_o = 3$ inches is given by

$$P_3 = \sqrt{\frac{1}{3}} \times P_1 = - 248,280 \text{ psi}$$

A. CASE 1

1. Loading Wave

From equation (28) the density ratio at $\bar{t} = 0$ is given by

$$\zeta = \frac{\rho_o}{\rho} = \frac{C_o}{C_o - u} = \frac{2.00 \times 10^5}{2.00 \times 10^5 - 2864.6} = .98568$$

The maximum σ_1 at the start of plastic deformation is given by

$$\sigma_1 = \frac{\sigma_y}{1-\mu} = - \frac{250,000}{0.7} = - 357,143 \text{ psi}$$

where $\sigma_y = - 250,000$ psi

$\mu = 0.7$

Thus $\zeta = 1 - \frac{357,143}{30 \times 10^6} (1-2x0.3^2) = 1 - .0119 \times .82 = .99042$

at $\sigma_y = - 357,143$ psi

As soon as the material begins stretching out, the loading becomes plastic. Considering $\bar{E} = 0$ from equation (38), we have

$$w \quad \sqrt{\bar{\sigma}_y \cdot \zeta} \cdot \sqrt{.00833 \times .98568} = \sqrt{.0082106} = .09061$$

where $\quad \bar{\sigma}_y = \dfrac{250,000}{30 \times 10^6} = .00833$

Also, $\quad \bar{u}_1 = \dfrac{u}{C_o} = -\dfrac{2864.6}{2 \times 10^5} = -.01432$

Substituting in equation (39), we have

$$\mathscr{E} = \cosh .09061\bar{t} - \frac{.01432}{.09061} \sinh .09061\bar{t} - 1$$

Since $\quad \bar{u} = \dfrac{d\mathscr{E}}{dE}$, we have

$$\bar{u} = .09061 \{\sinh .09061\bar{t} - .15800 \cosh .09061\bar{t}\}$$

First, the time $\bar{t}_o$, where $\bar{u} = 0$, is found by plotting $\bar{u}$ versus $\bar{t}$. (See Figure 2). The $\mathscr{E}$ versus $\bar{t}$ plot is shown in Figure 3: it is shown that as $\bar{u}$ approaches zero, $\mathscr{E}$ approaches a constant value of 0.0125. The radial strain is given by $\epsilon_1 = \zeta \dfrac{r_o}{r} - 1$: The ϵ_1 versus $\bar{t}$ plot is shown in Figure 4: ϵ_1 approaches the value of .0264.

2. Unloading Wave

The duration of the triangular wave is 20 μsec: its equivalent rectangular profile will have a duration of 10 μsec. Hence,

$$\bar{t} = \frac{C_o t}{r_o} = \frac{2.0 \times 10^5 \times 10 \times 10^{-6}}{1} = 2.0$$

It is shown in Figure 2, however, that the loading process terminates at $\bar{t} = 1.75$. Thus, when the unloading wave occurs, the material is in equilibrium.

For the unloading wave, we have the following constants (equation (53)):

$\bar{\bar{E}} = 1$

$\zeta = 1$: The material returns to its initial volume.

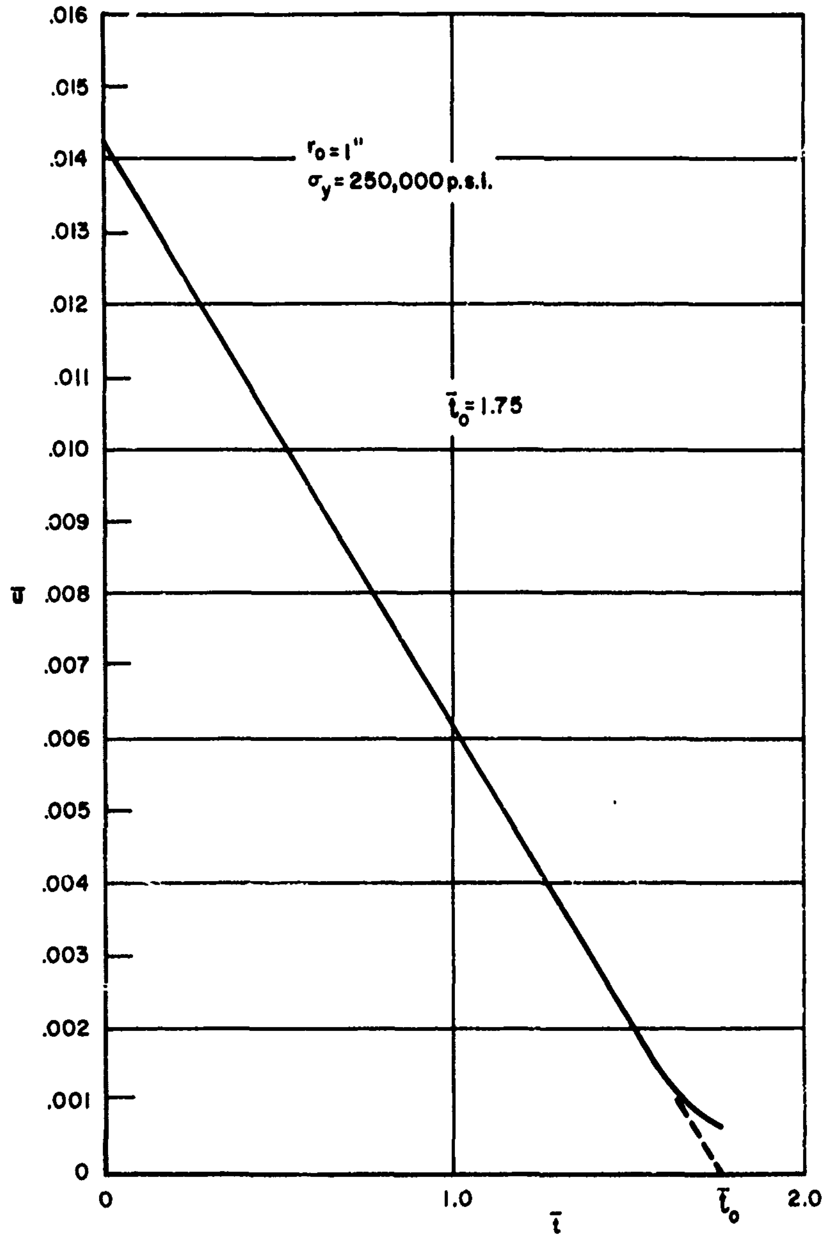
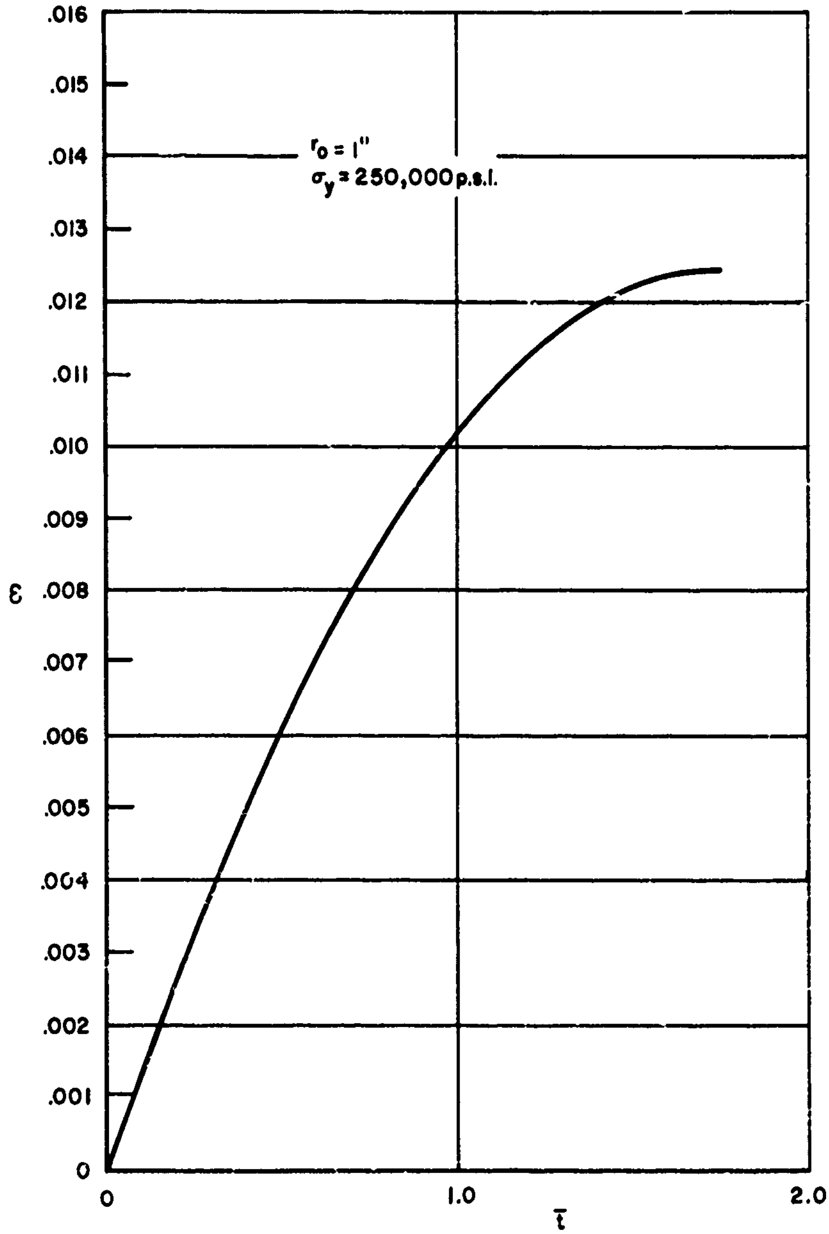

.016
.015
.014
.013
.012
.011
.010
.009
.008
.007
.006
.005
.004
.003
.002
.001
0
$\bar{u}$
$r_0 = 1''$
$\sigma_y = 250{,}000$ p.s.i.
$\bar{t}_0 = 1.75$
0
1.0
2.0
$\bar{t}_0$
$\bar{t}$

FIGURE 2


.016
.015
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.006
.005
.004
.003
.002
.001
0
ε
$r_o = 1''$
$\sigma_y = 250{,}000$ p.s.i.
0
1.0
2.0
$\bar{t}$

FIGURE 3

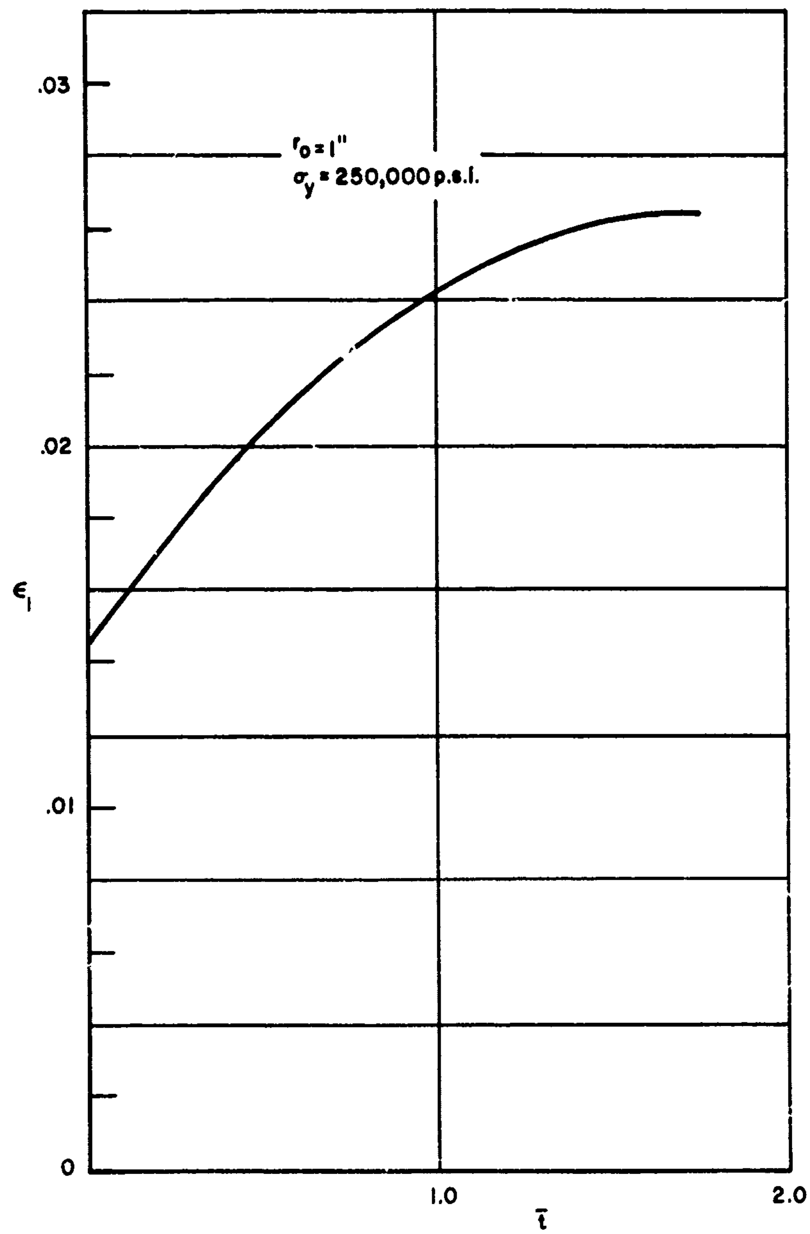


FIGURE 4

Hence $\alpha = \frac{1}{2} + \mu = 0.5 + 0.303 = .803$

where $\mu = 0.303$. N.B. The value of σ_1 after unloading is zero

$\beta = 1 - \mu = 0.697$

$\gamma = 0.303$

$u_o = .01432$

$\xi = - 1.0000 - \bar{u}_o = - 1.0000 - .01432 = - 1.01432$

$\psi = \sqrt{\frac{\alpha^2}{4} + 4\beta} = 1.71697$

$\frac{\psi}{2} = .8585$

Thus $\frac{\gamma}{\beta} = \frac{.303}{0.697} = .43472$ and $\frac{\alpha}{2} = 0.4015$

Hence $\xi + \frac{\alpha}{2} (\frac{\gamma}{\beta}) = - 1.01432 + 0.4015 (.43472)$

$= - 1.01432 + .17454 = - .83978$

and $Y = \frac{\xi + \frac{\alpha}{2} (\frac{\gamma}{\beta})}{\frac{\psi}{2}} = - \frac{.83978}{.8585} = - .978195$

Also, $\frac{\psi}{2} \times \frac{\gamma}{\beta} = .8585 \times .43472 = .37321$

$\frac{1}{2} \times \frac{\gamma}{\beta} = .43472 \times .5 = .21736$

$\frac{\alpha}{2} Y = - .4015 \times .978195 = - .392745$

$\frac{\gamma}{\beta} \times \frac{\alpha}{2} = .43472 \times .4015 = .17454$

$\frac{Y}{2} = - \frac{.978195}{2} = - .489097$

and $\frac{1}{2} \frac{\gamma}{\beta} - 1 = - 1 + .21736 = - .78264$

Hence $\bar{u} = .27686 \sinh .8585\bar{t} - .79696 \cosh .8585\bar{t} + .78264 e^{.4015\bar{t}}$

and $\mathcal{E} = e^{-.4015\bar{t}} \{.43472 \cosh .8585\bar{t} - 1.01432 \sinh .8585\bar{t}\} - .43472$

In the plot of $\bar{u}$ versus $\bar{t}$, Figure 5, the time when the velocity becomes zero is shown as $\bar{t}_o = 0.026$ ($t = .26 \times 10^{-6}$ sec). Substituting in the equation of $\mathcal{E}$ we find the residual hoop strain, $\mathcal{E} = .01713$. Equations (51) and (52) were derived, however, with $\mathcal{E}_o = 0$ at $\bar{t} = 0$ (with reference to the equilibrium conditions after the loading).

To find the residual hoop strain after unloading, we have:

$$\Delta \mathcal{E} = \mathcal{E}_{unloading} - \mathcal{E}_{loading}$$

$$= .01713 - .0125 = \underline{.00463} \text{ (compression)}$$

In order to find the residual hoop stress we use the principle of elastic recovery. Thus, the volumetric strain e_v is given by:

$$e_v = \epsilon_1 + \mathcal{E}_{unloading}$$

where ϵ_1 is the radial linear strain change upon relief and $\mathcal{E}$ is the corresponding hoop strain. We know, however, the radial unloading stress: i.e., $\Delta\sigma_1 = 430{,}000$ psi. Hence, $\epsilon_1 = \frac{430{,}000}{30 \times 10^6} = .0143$.

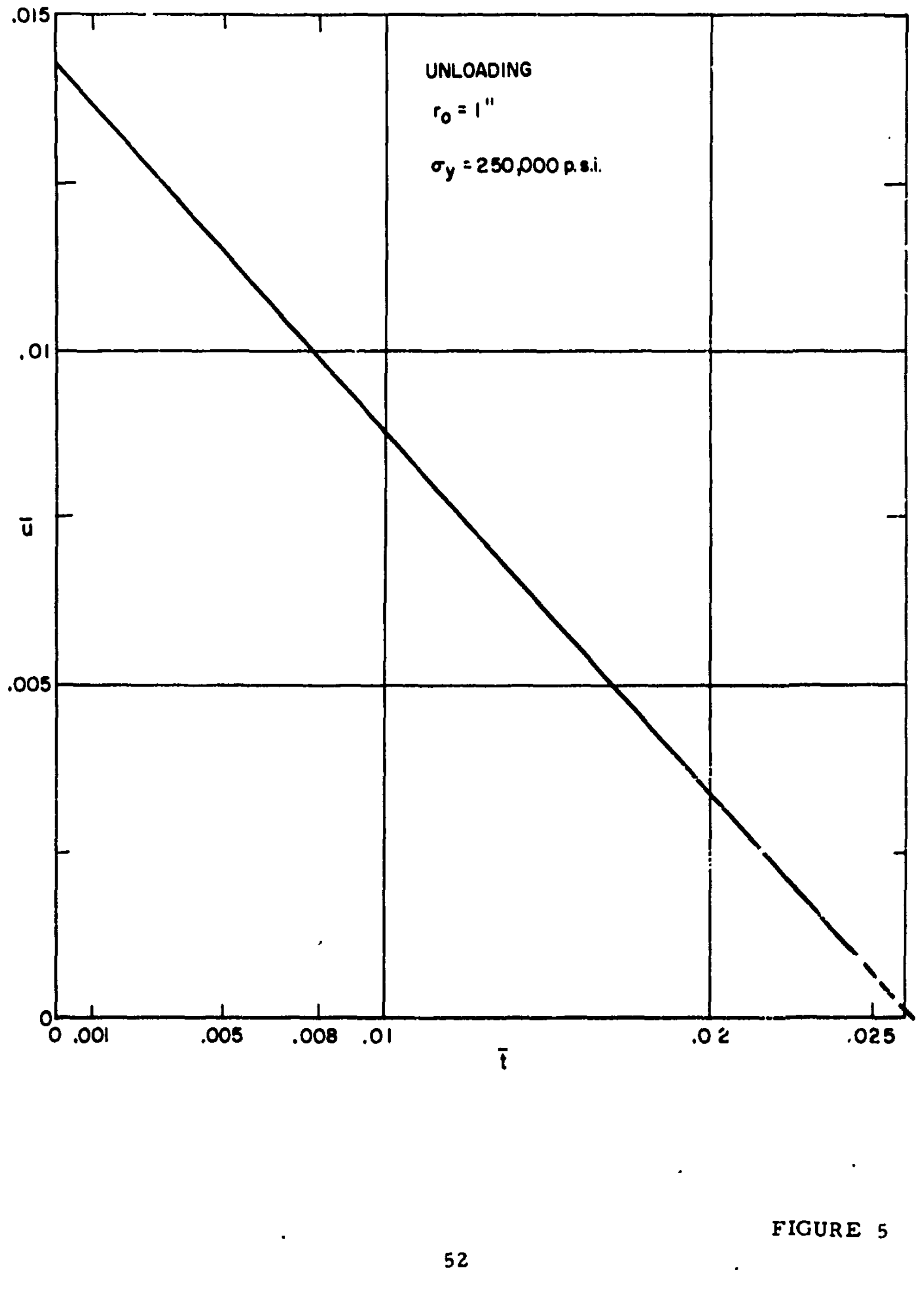
The value of e_v is also known from the loading conditions:

$$-e_v = \frac{\Delta \bar{v}}{V_o} = -\frac{\rho}{\rho_o} + 1 = +.01813$$

Thus, the hoop strain for zero stress (permanent set) is given by

$$\mathcal{E}_o = e_v - \epsilon_1 = .01813 - .0143 = \underline{.0036}.$$

The corresponding permanent set in the radial direction is $\epsilon_o = .026 - .0143 = \underline{.012}$. In the calculations for the unloading wave, it was assumed that upon relief the recovery was instantaneous: thus $\frac{r}{r_o} = 1$ at $t = 0$.



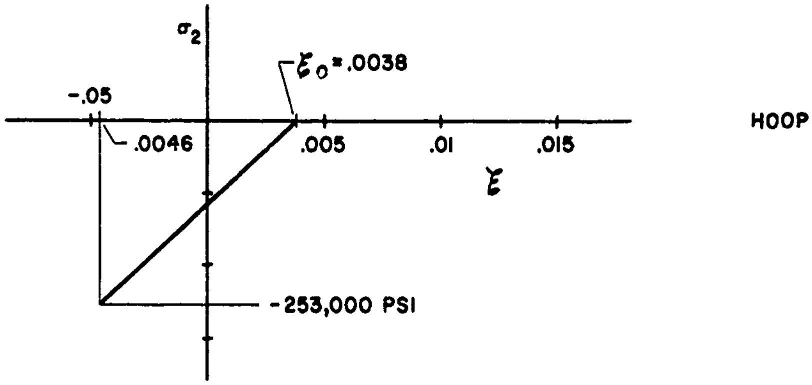
UNLOADING
$r_0 = 1''$
$\sigma_y = 250{,}000$ p.s.i.
.015
.01
.005
0
$\bar{u}$
0 .001
.005
.008
.01
.02
.025
$\bar{t}$

FIGURE 5

Then, for $\bar{t} > 0$, $\frac{r}{r_o}$ decreases, changing both hoop and radial strains according to to the following equations:

$$\xi = 1 - \frac{r}{r_o} \quad \text{and} \quad \epsilon_1 = \zeta \frac{r_o}{r} - 1.$$

Hence, the material will be under the following strain conditions :



Without pursuing this case any further, it is easily concluded that secondary yielding will take place. This is undesirable for most designs.

B. CASE 2

1. Loading Wave

The velocity at r_o = 3 inches was given as:

$$u_3 = -1654.0 \text{ and } \bar{u}_3 = \bar{u}_o = -\frac{1654}{2 \times 10^5} = -\underline{\underline{.00827}}$$

$$\zeta = \frac{C_o - u}{C_o} = 1 - \bar{u}_o = .99173, \text{ and } \sigma_{1(\text{elastic})} = \frac{250,000}{0.7} = 357,143 \text{ psi}$$

Hence, $\omega = \sqrt{.00833 \times .99173} = \sqrt{.0082611} = .0908906$

$$\therefore \quad \frac{u_o}{\omega} = - \frac{.00827}{.0908906} = -.0909885$$

Hence $\quad \mathcal{E} = \cosh .0908906\,\bar{t} - .0909885 \sinh .0908906\,\bar{t} - 1$

and $\quad \bar{u} = .0908906 \{\sinh .0908906\,\bar{t} - .0909885 \cosh .0908906\}$

The duration of the wave, on the other hand, is given by

$$\bar{t} = \frac{10 \times 10^{-6} \times 2 \times 10^5}{3} = \frac{2}{3}$$

where

$t = 10 \times 10^{-6}$

$C_o = 2 \times 10^5$

$r_o = 3$ inches

The $\bar{u}$ versus $\bar{t}$ and $\mathcal{E}$ versus $\bar{t}$ plots are shown in Figures 6 and 7, respectively. Another case of $\sigma_y = 150,000$ psi is also shown for comparison.

With the value of $\bar{t}$ at $r_o = 3$ inches being smaller than at $r_o = 1$ inch, it is shown that the unloading wave occurs before the material achieves equilibrium.

2. Unloading Wave

Following the same procedure as for $r_o = 3$ inches, with

$\bar{u}_o = 0.00827 - .00275^* = .00552$

$\mathcal{E} = e^{-.4015\,\bar{t}} \{.43472 \cosh .8585\,\bar{t} - .96794 \sinh .8585\,\bar{t}\} - .43472$

and $\quad u = -.27793 \sinh .8585\,\bar{t} + .78816 \cosh .8585\,\bar{t} - .78264\, c^{.4015\,\bar{t}}$

The plot of $\bar{u}$ versus $\bar{t}$ is shown in Figure 8. Comparing this unloading with that of $r_o = 1$ inch, we see that, at $r_o = 3$ inches, equilibrium is achieved in about half the time.

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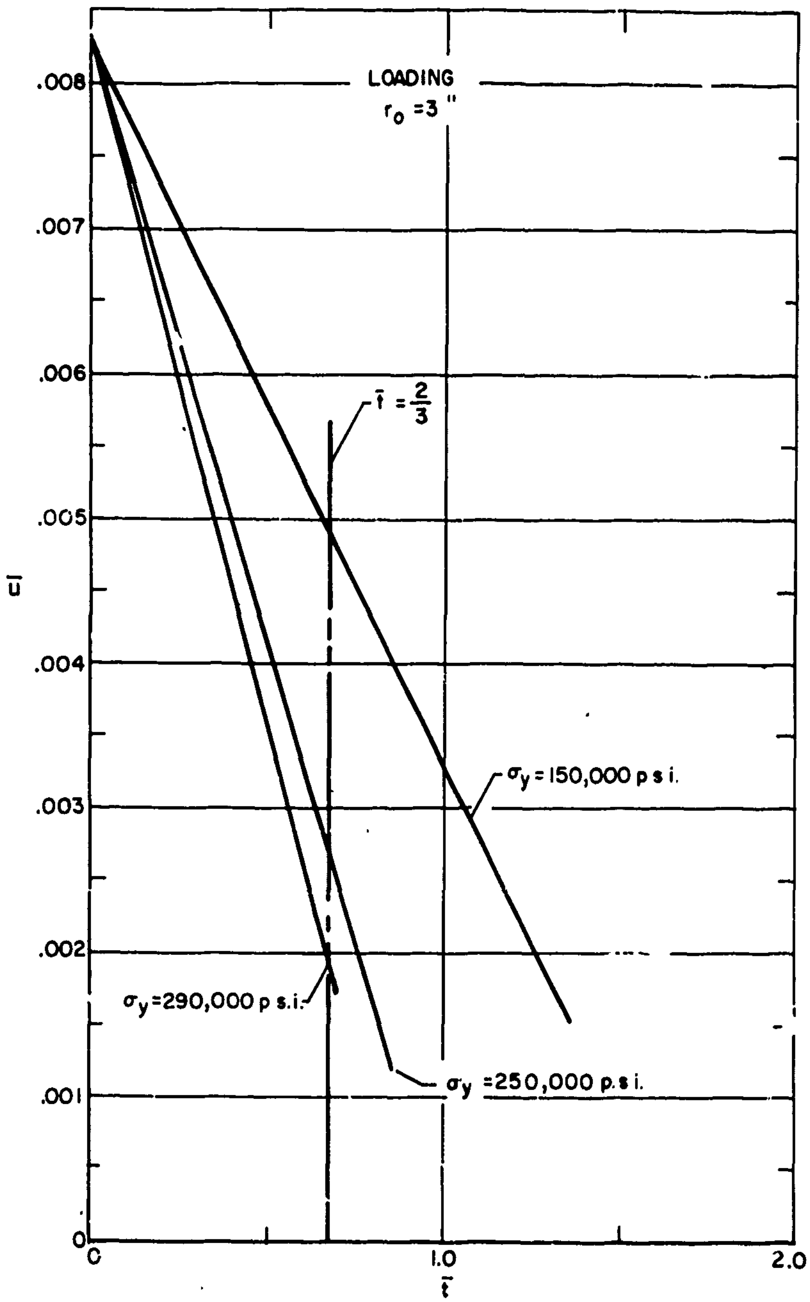
* velocity from the loading process



LOADING
$r_o = 3''$
$\bar{t} = \frac{2}{3}$
$\sigma_y = 150{,}000$ p s i.
$\sigma_y = 290{,}000$ p s.i.
$\sigma_y = 250{,}000$ p.s i.
$\bar{u}$
$\bar{t}$
.008
.007
.006
.005
.004
.003
.002
.001
0
0
1.0
2.0

FIGURE 6

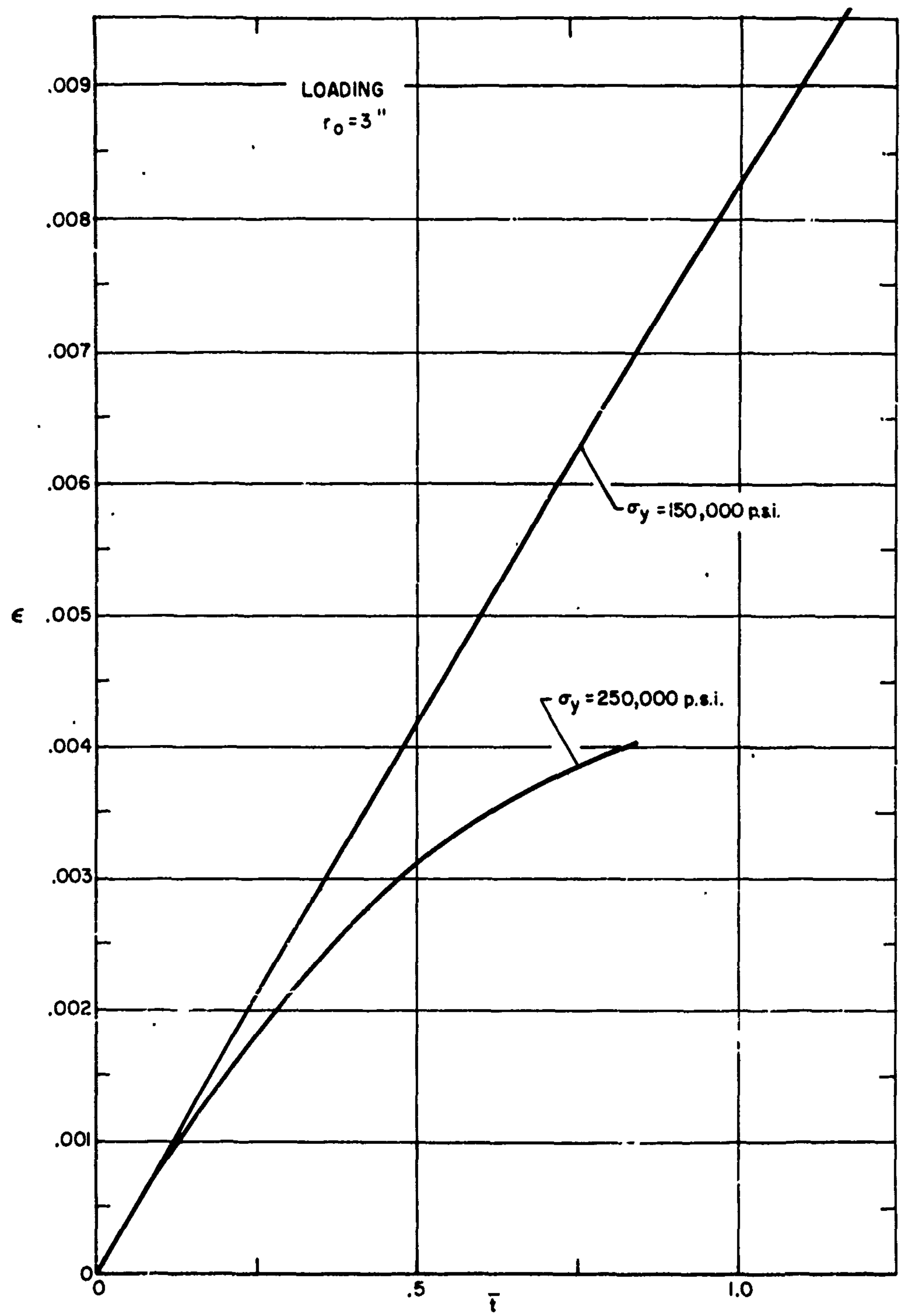

LOADING
$r_o = 3''$
$\sigma_y = 150{,}000$ p.s.i.
$\sigma_y = 250{,}000$ p.s.i.
ϵ
.009
.008
.007
.006
.005
.004
.003
.002
.001
0
0
.5
1.0
$\bar{t}$

FIGURE 7

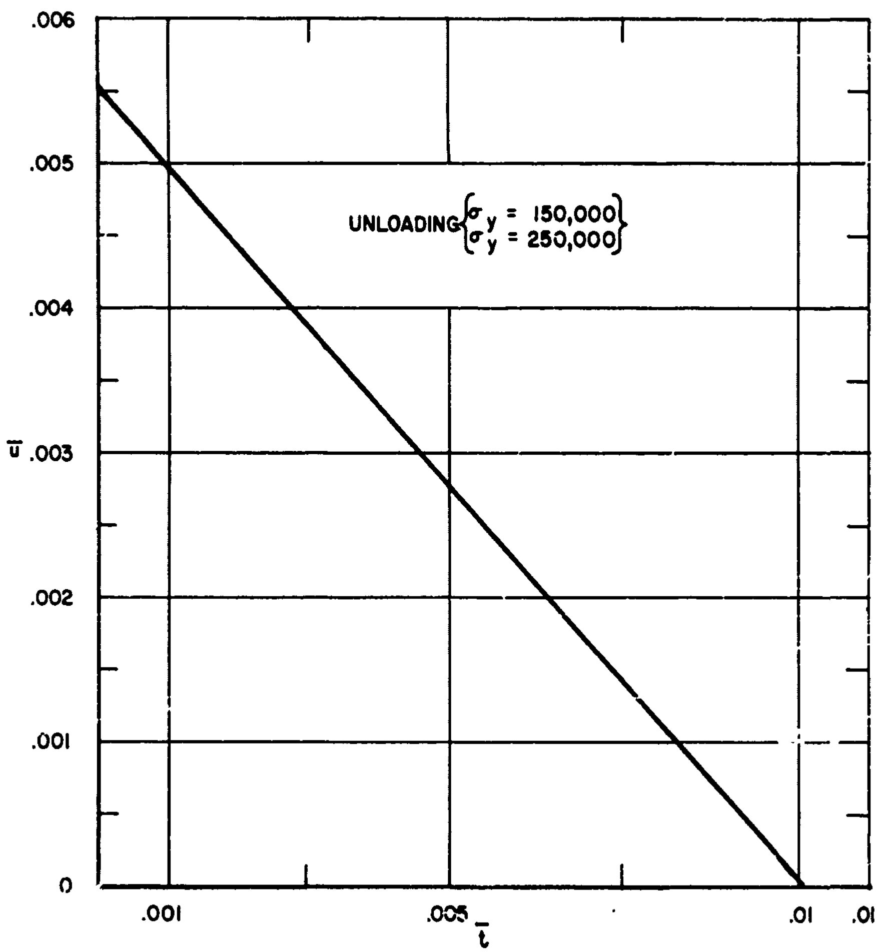

.006
.005
.004
$\bar{u}$ .003
.002
.001
0
.001
.005 $\bar{t}$
.01
.011
UNLOADING $\begin{cases}\sigma_y = 150{,}000\\ \sigma_y = 250{,}000\end{cases}$

FIGURE 8

The corresponding value of $\mathcal{E}$ at $\bar{t} = .01$ is $\mathcal{E} = .00727$. The $\mathcal{E}$ value at $\bar{t} = 0.667$ for loading is $\mathcal{E} = .00365$. Hence, the resulting strain is (.00733 - .00365) or $\mathcal{E} = \underline{.00368}$ (compression).

To find the resulting stresses, the permanent set in both the hoop and radial directions must be found. Employing the concept of elastic recovery again, we have

$$e_v = \mathcal{E}_o + \epsilon_1$$

$$\epsilon_1 = \frac{248,280}{30 \times 10^6} = .008276$$

Furthermore, $\quad - e_v = + \dfrac{\Delta V}{V_o} = - \dfrac{\rho}{\rho_o} + 1 = - .99173 + 1 = .00827.$

Thus, $\mathcal{E}_o = 0$: This result indicates that there is no permanent set in the hoop direction. In the radial direction, $\dfrac{r}{r_o} = 1 + \mathcal{E} = 1 + .00365 = 1.00365$ from loading. Hence $\dfrac{r_o}{r} = \dfrac{1}{1.00365} = .99636$. Also, $\dfrac{\rho_o}{\rho} = .99173$.

$$\therefore \qquad \epsilon_1 = .99636 \times .99173 - 1 = .98812 - 1 = \underline{.01188} \text{ from loading.}$$

During the unloading, there is the elastic recovery where $\epsilon_1 = .00827$: then, there is the further change of ϵ_1 due to the $\dfrac{r}{r_o}$ change.

Hence $\qquad \epsilon_1 = \dfrac{r_o}{r} - 1 + .00827$

$$\frac{r}{r_o} = 1 - \mathcal{E}' = 1 - .00727 = .99373$$

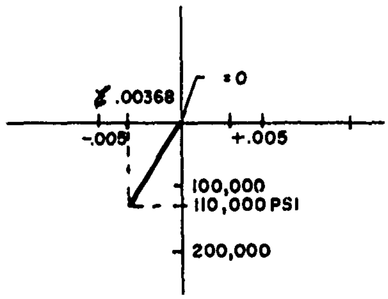
$$\therefore \qquad \frac{r_o}{r} = 1.0063$$

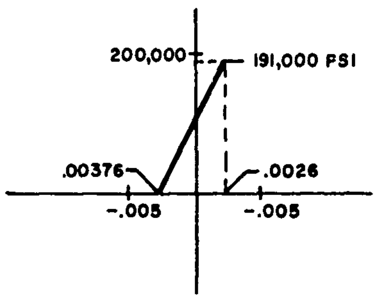
and $\qquad \epsilon_1 = .0063 + .00834 = .01464$

Thus, change in ϵ_1 = .01464 - .0119 = .00274 (tension). The value of ϵ_1 when the radial stress is zero is .01203 - .00827 = .00376.

Residual radial stress = .00376 + .0026 = .00636 or .00636 x 3 x 10^6 = 190,800 psi (tension).

The residual hoop stress = .00368 x 3 x 10^6 = 110,400 psi (compression).





From the secondary yielding criterion, we have

$$\sigma_2 - \sigma_1 \leqslant \sigma_y$$

or 191,000 + 110,000 = 301,000 psi. Since σ_y was assumed at 250,000 psi, the material will go through secondary yielding.

SECTION VII - DISCUSSION

The theory of unsteady waves was used to develop working equations for computing the radial and hoop strains in hollow cylinders under transient internal loads. The wa[illegible] propagates with a steep elastic front which is followed by an attenuated plastic wave. The changes of density and velocity immediately behind the front are instantaneous: furthermore, the plastic wave is not attenuated at $t=0$. From this pattern it is easily seen why the stress field of the material at $t=0$ is three-dimensional and the shear stress considerably higher than for slowly applied loads. Under hydrostatic pressure, at $t=0$, the hoop stress is compressive (for a compressive wave). As soon as the deceleration process starts, however $(t>0)$, the material is under tension in the hoop direction. The change from compressive to tensile hoop stress does not, however, take place instantaneously. Thus, the stress behind the wave remains elastically higher than when the load is applied slowly until the transition from compressive to tensile stress is made. This may be the reason why higher yield stresses are manifested during impulsive loads.

Another important aspect of the analysis is the duration of the wave as it propagates through the wall of the cylinder. The dimensionless parameter $\bar{t} = \frac{tC_o}{r_o}$ is constant for all r_o. Thus, as r_o increases, the time t to achieve steady state also increases. The results are shown in the example where the cases for $r_o = 1$ and $r_o = 3$ are analyzed. At $r_o = 1$, the material will experience plastic flow first in the outward direction and then in the inward direction (secondary yielding): at $r_o = 3$, the same events take place but to a lesser degree. If, however, the σ_y had the value of 290,000 psi the material could withstand the pressure at $r_o = 3$ without secondary yielding. Thus, after the first pulse the material will withstand the loads for all subsequent pulses without further yielding.

In his tests with thick-walled cylinders, Rinehart[4] has shown that the mode of failure near the bore was in shear while farther out from the bore the material fractured in tensile hoop stress. This can be attributed mainly to the high hydrostatic pressure at $t = 0$: as the wave amplitude decreases, the hydrostatic pressure decreases, allowing the material to be predominantly in hoop tension.

CONCLUSIONS

1. Extrusion of an appropriate billet from an ultra high hydrostatic fluid pressure medium (preferably inhibited water) is theoretically possible from the effects of a superposition of a detonation shock type pressure pulse created from the rapid discharge of capacitor stored energy across a suitable gap.

2. As the relationship $u_0^* = \frac{8\,fy}{\rho_0} \ln \frac{r_0}{r}$ derived from the variables associated with electrohydraulic extrusion bears a striking similarity to the well known empirical expression for extrusion, $P = \sigma \ln \frac{A_0}{A}$ where P = pressure to extrude, σ = resistance to deformation and $\ln \frac{A_0}{A}$ = log of the extrusion ratio, a logical analytic approach to the proposed extrusion experiments is available.

3. Extrusion for one pulse is dependent upon the shock pressure pulse amplitude. The final extrusion length will be dependent upon the rate of pulses generated from a series of capacitor discharge events.

4. A pressure vessel can be designed to contain a 200, 000 psi hydrostatic fluid pressure with a superposed dynamic detonation pressure pulse loading of 425, 000 psi after reflection.

5. At hydrostatic pressures of 200, 000 psi, the particle velocity in the walls of the container as a consequence of the capacitor discharge event attenuates rapidly from a critical value to a value, which considered with the static stress pattern is within the elastic property limits of accepted container design materials.

6. From the pressurized fluid boundary outward into the container walls for a distance of 2 inches, the behavior of the metal is critical because of high particle velocity and cyclic loading conditions. For this reason, the pressure vessel design must include a replaceable

liner of ultra high strength material. This liner must be put into a condition of positive axial and radial restraint.

FUTURE WORK

A purchase order based on analysis of vendor proposals will be issued by Republic Aviation Corporation for the procurement of the pressure vessel and pumping system best suited to the objectives of the program.

The derived equations involving the radial propagation of stress waves in thick-walled cylinders under transient detonation internal loads along with conventional static stress criteria, and extrusion experimental requirements will be applied to the electro-hydraulic pressure vessel design problem to formulate a final optimum working design. From these engineering considerations, assembly drawings will be produced by the selected vendor.

Upon approval by Republic Aviation, the assembly drawings will be worked into detailed design drawings suitable for manufacture of the equipment. Fabrication of the pumping equipment will commence.

An investigation into presently attainable charging rates and methods of rapid cyclical release of high voltage capacitor discharge energy will be made to obtain a practical, economical approach to the equipment best suited to the requirements of electrohydraulic extrusion.

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